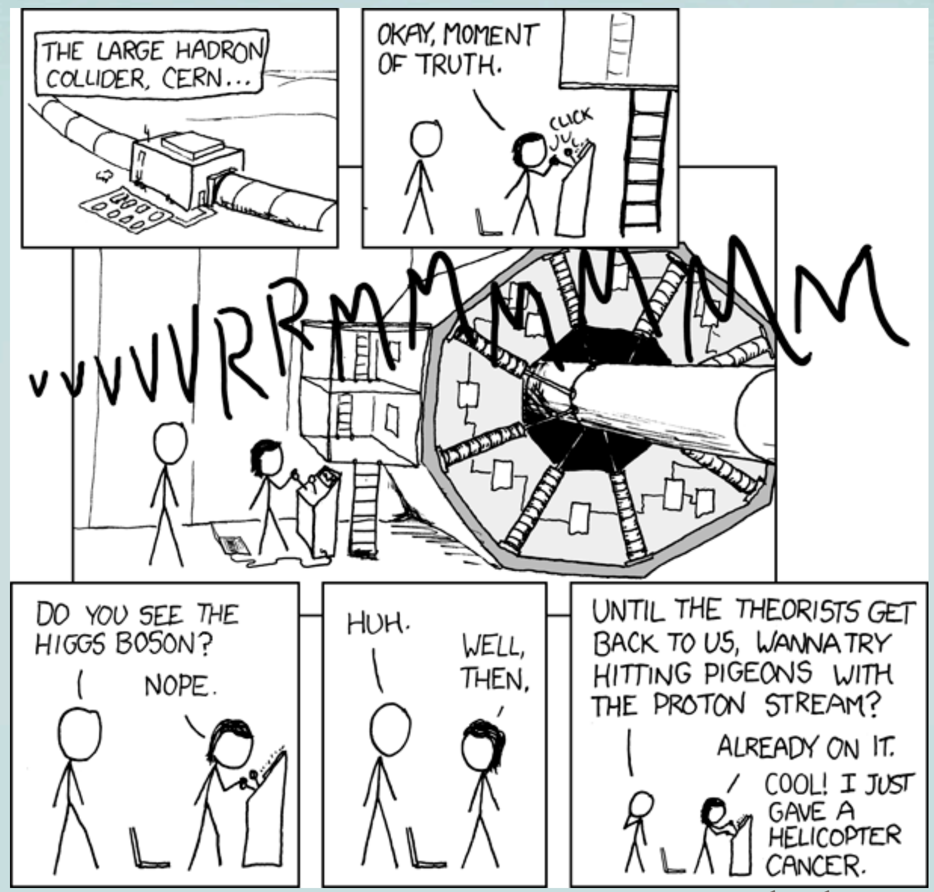
PERTURBATIVE QCD FOR LHC PHYSICS



xkcd.com

XXXVII SLAC Summer Institute: Revolutions on the Horizon

August 7, 2009

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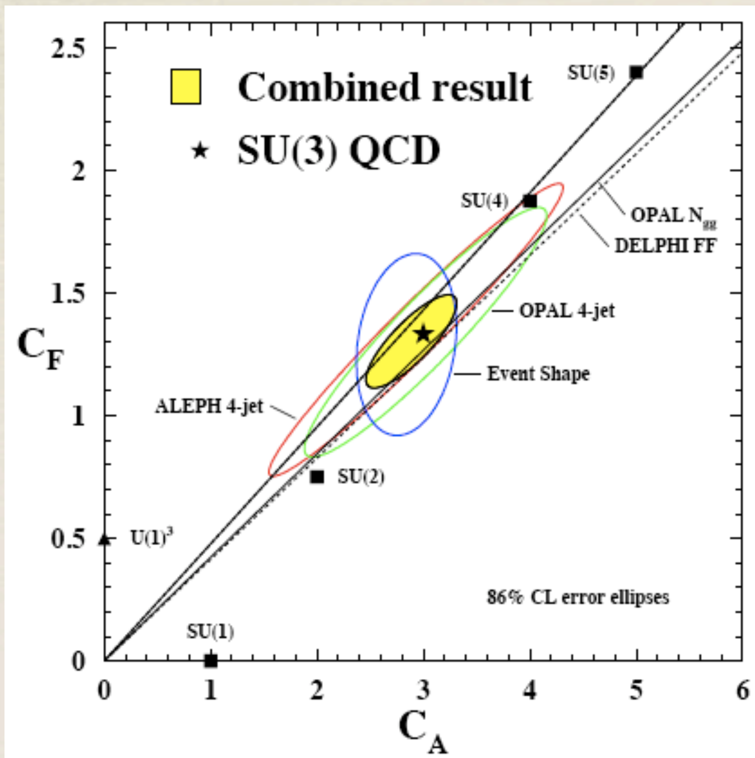
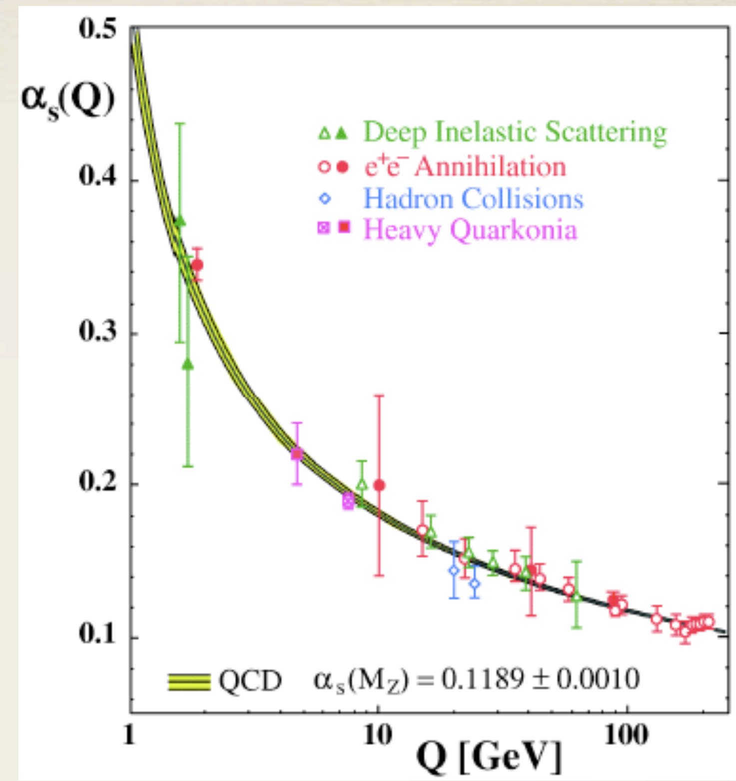
Outline

- Can't possibly tell you all the important issues in one hour... will attempt to mention most things, do 1-2 in detail (and go over time...)
- Structure of QCD: factorization and universality
- Partonic cross sections: leading-order (LO), NLO, NNLO
- Matching fixed-order calculations with parton showers
- Parton distribution functions (PDFs) and their errors
- Omissions: jet algorithms (G. Salam, 0801.0070), resummation (G. Sterman, hep-ph/0412013)

Status of pQCD





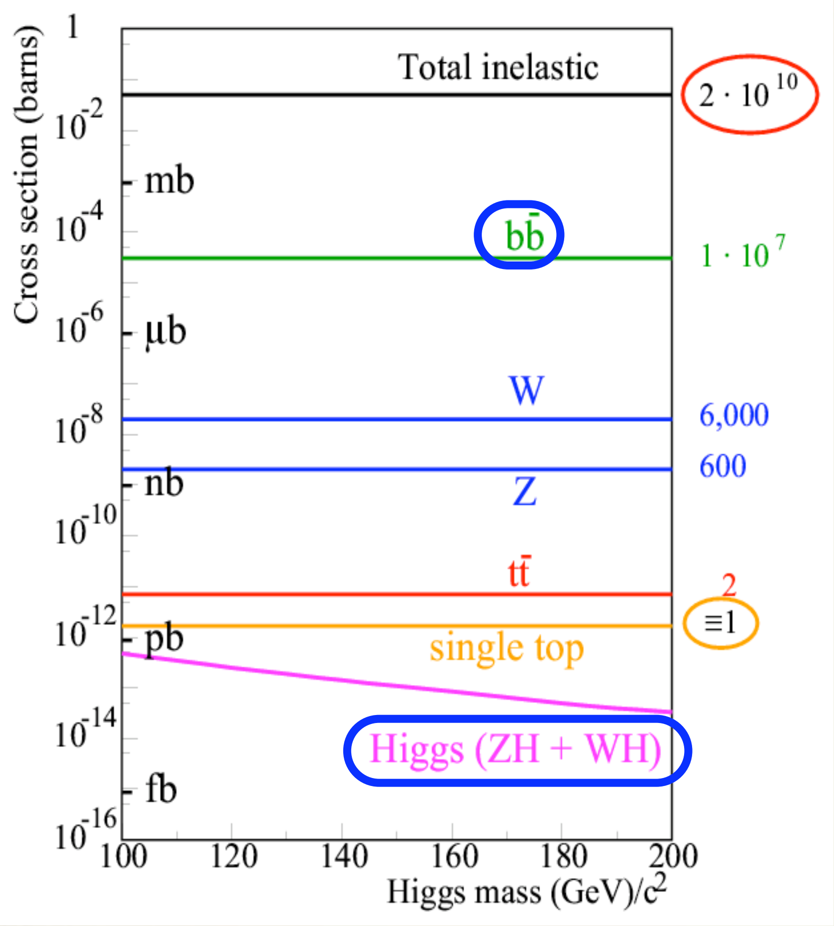
SU(3) gauge theory of QCD established as theory of Nature

Predicted running of α_s established in numerous experiments over several orders of magnitude

2004: Gross, Politzer, Wilczek

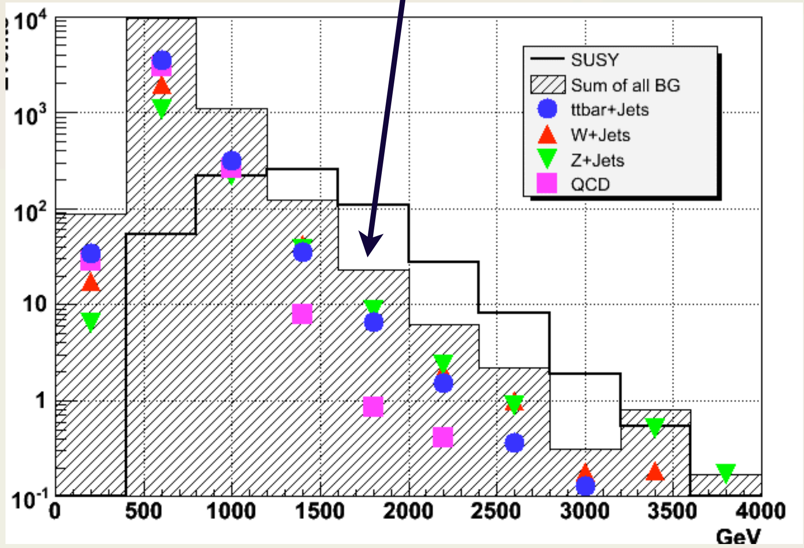
Why do we still care about QCD?

The revolution crushed

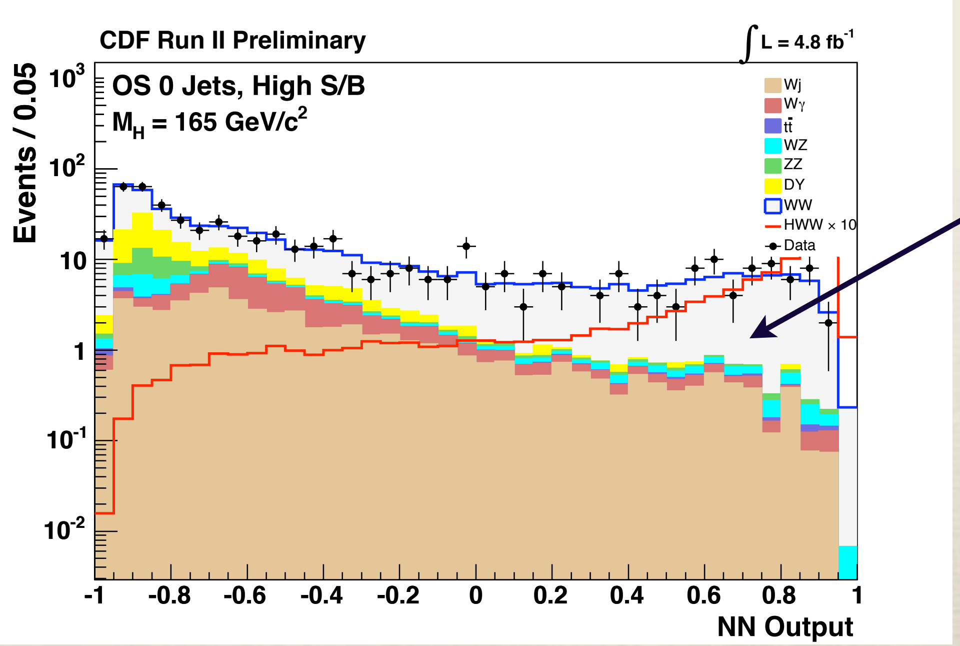


Enormous challenge to understand signal, background to be sure of discovery!

Do we understand the QCD shape prediction for W/Z+jets?

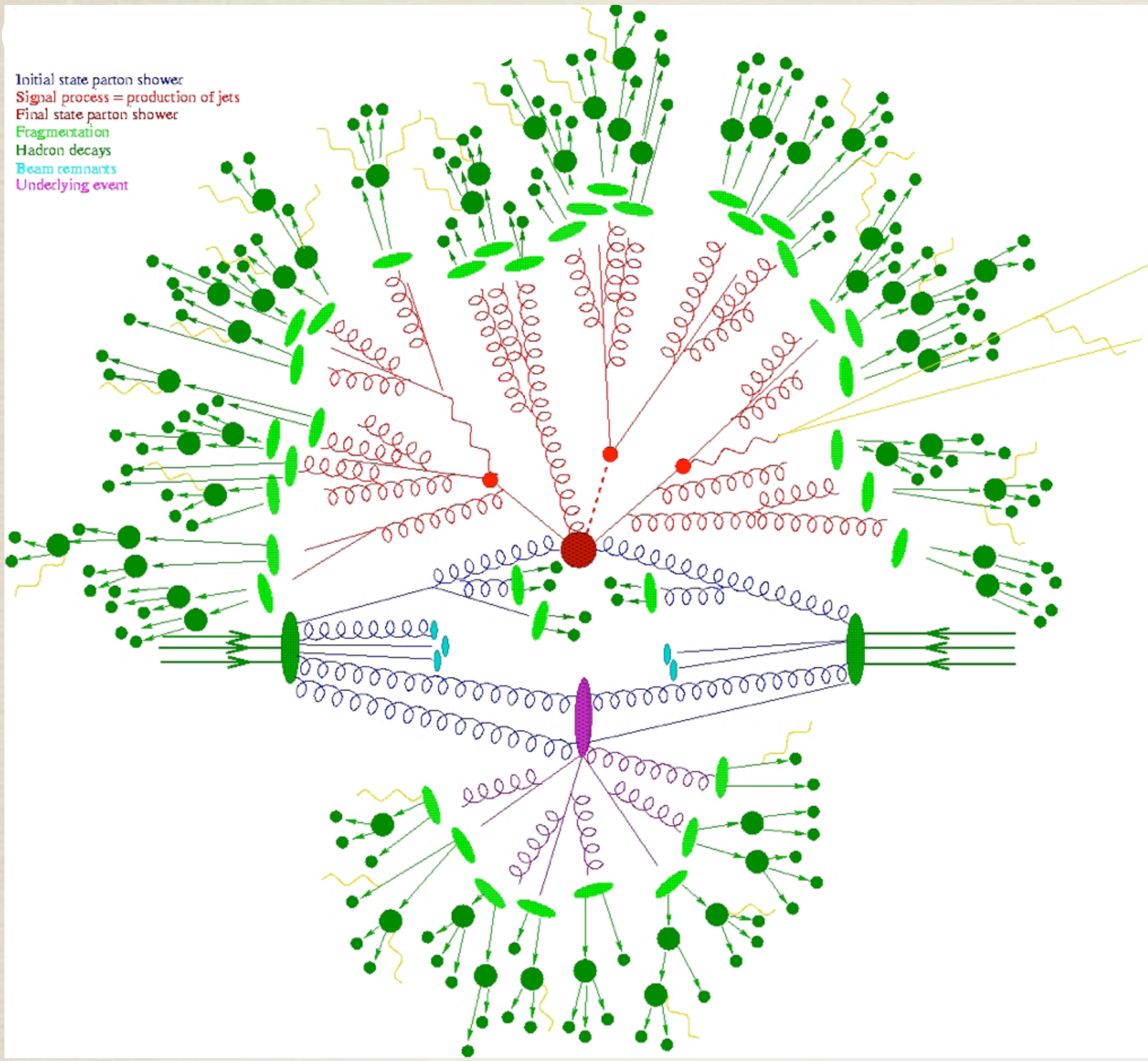


ATLAS TDR: S/B >10
Current: S/B~2



What is the QCD prediction for the di-boson production rate?

Collisions at the LHC



A lot going on...

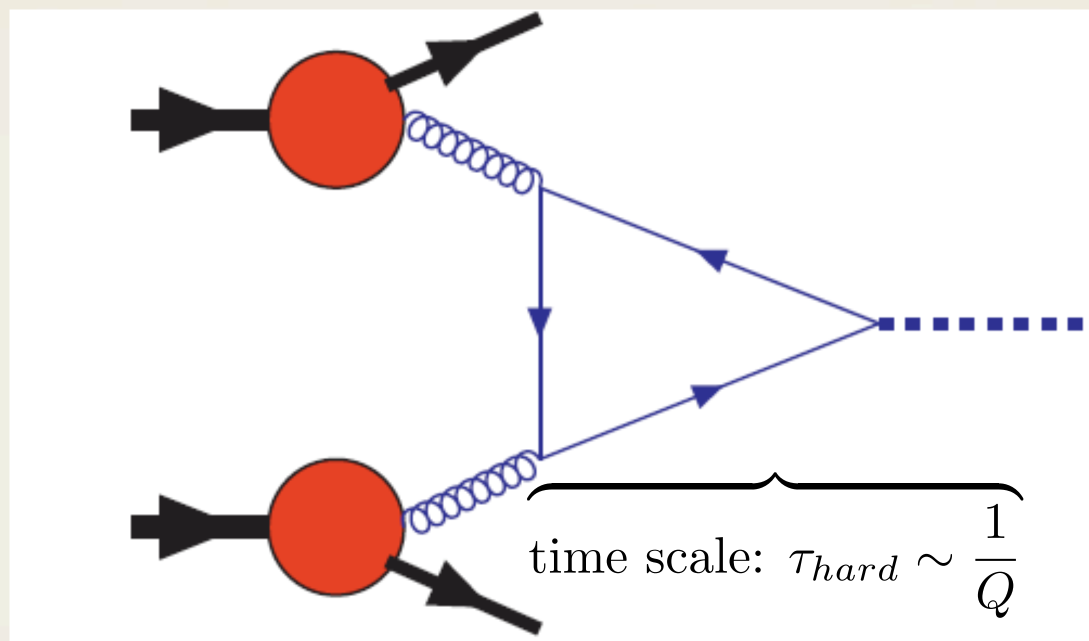
- New physics at hard scale; M_H for example
- Parton shower evolution from M_H to Λ_{QCD}
- Final state hadronization at Λ_{QCD}
- Parton distribution functions at Λ_{QCD}
- Multiple parton interactions, hadron decays, ...

How does one make a prediction for such an event?

Divide and conquer: PDFs

Make sense of this with *factorization*: separate hard and soft scales

$$\overbrace{\text{time scale: } \tau_{proton} \sim \frac{1}{\Lambda_{QCD}}}$$



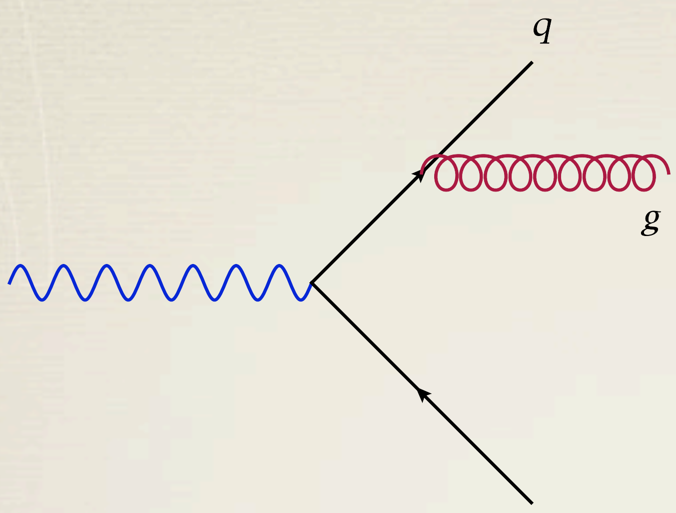
$$\sigma_{h_1 h_2 \to X} = \int dx_1 dx_2 \underbrace{f_{h_1/i}(x_1; \overbrace{\mu_F^2}^{\text{factorization scale}}) f_{h_1/j}(x_2; \mu_F^2)}_{PDFs} \underbrace{\sigma_{ij \to X}(x_1, x_2, \mu_F^2, \{q_k\})}_{\text{partonic cross section}} + \underbrace{\mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)^n}_{\text{power corrections}}$$

Non-perturbative but *universal*; measure in DIS, fixed-target, apply to Tevatron, LHC

Process dependent but calculable in pQCD

Small for sufficiently inclusive observables

Parton shower evolution



Factorization in limit of collinear gluon emission

$$\begin{aligned} d\sigma_{n+1} &\rightarrow d\sigma_n \frac{\alpha_s}{2\pi} \overbrace{\frac{d\theta}{\theta}}^{\text{collinear sing.}} P_{q\to q}(z)\, dz \\ P_{q\to q}(z) &= C_F \frac{1+z^2}{1-z} \quad \text{(Altarelli-Parisi splitting function)} \\ z &= \text{Energy fraction of quark} \end{aligned}$$

Multiple emissions exponentiate to give *Sudakov form factor* ⇒ universal

$$S(t) = \exp\left\{ -\int_{t_0}^{t} dt \int_{z_-(t)}^{z_+(t)} dz \frac{\alpha_s}{2\pi} P_{q\to q}(z) \right\}, \quad \underbrace{t = p_T^2, E^2\theta^2, \ldots}_{\text{ordering variable}}$$

→ Probablity of no emission

Evolve each parton in t using S(t) until lower cutoff reached

Probability of emission: $\frac{\alpha_s}{\pi} \ln^2 \frac{\hat{s}}{\Lambda^2_{QCD}} \approx 1$ LHC events very 'jetty'

Recipe for a QCD prediction

- Calculate $\sigma_{ij \to X}$
- Evolve initial, final states to Λ_{QCD} using parton shower
- Connect initial state to PDFs, final state to hadronization

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How precisely must we know σ?

Do we know how to combine σ, parton shower?

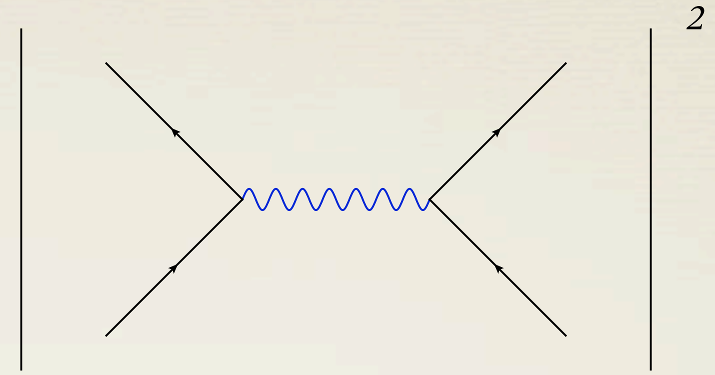
Are our observables inclusive (e.g., lepton η) so we can avoid a parton shower?

Do we have hard jets? Parton showers assume soft/ collinear radiation

Do we know the PDFs in the relevant kinematic regions?

Computing σ: LO

$$\sigma = \overbrace{\sigma_0}^{LO} + \overbrace{\frac{\alpha_s}{\pi}\sigma_1}^{NLO} + \overbrace{\left(\frac{\alpha_s}{\pi}\right)^2 \sigma_2}^{NNLO} + \ldots$$



+ Easy to calculate; codes have automated this in the SM and beyond (ALPGEN, MADGRAPH, COMPHEP, ...)

+ Gets hard emissions and angular correlations correct (based on full QCD, unlike parton shower)

- Theoretical uncertainty large: μ_F, $\alpha_s(\mu_R)$ variation, often missing parametric dependences (gluon PDF in $qq \to l^+l^-$, for example)

W+jets

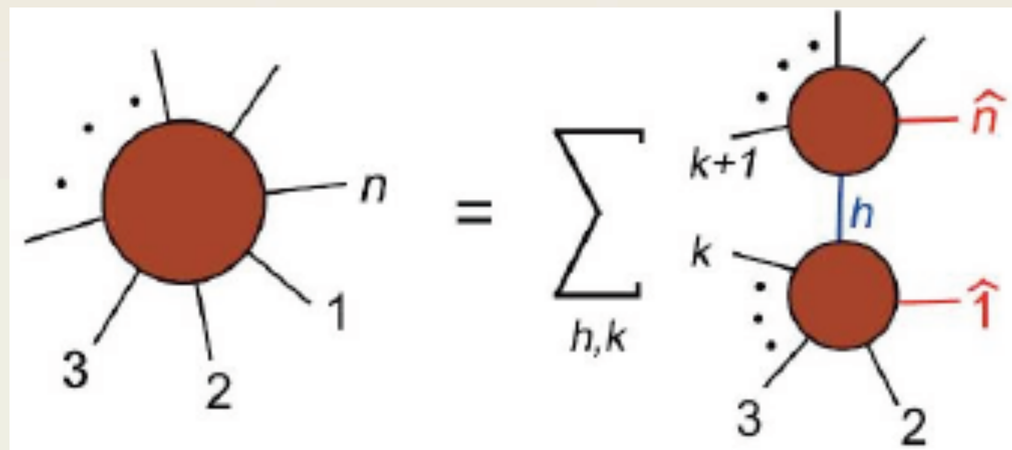
number of jets	CDF	LO
1	53.5 ± 5.6	$41.40(0.02)^{+7.59}_{-5.94}$
2	6.8 ± 1.1	$6.159(0.004)^{+2.41}_{-1.58}$
3	0.84 ± 0.24	$\boxed{0.796(0.001)^{+0.488}_{-0.276}}$

O(1) uncertainties in rate →

BLACKHAT: Berger et al., 0907.1984

Recursion relations

- Can go to high multiplicity at LO using *recursion relations* rather than diagrams (Berends-Giele, Cachazo-Svrcek-Witten, Britto-Cachazo-Feng)



$pp \to n$ jets gluons only	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
MC cross section [pb]	$8.915 \cdot 10^7$	$5.454 \cdot 10^6$	$1.150 \cdot 10^6$	$2.757 \cdot 10^5$	$7.95 \cdot 10^4$
stat. error	0.1%	0.1%	0.2%	0.5%	1%
	integration time for given stat. error [s]				
CSW (HAAG)	4	165	1681	12800	$2 \cdot 10^6$
CSW (CSI)	-	480	6500	11900	197000
AMEGIC (HAAG)	6	492	41400	-	-
COMIX (RPG)	159	5050	33000	38000	74000
COMIX (CSI)	-	780	6930	6800	12400

Feynman diagrams → AMEGIC (HAAG)

Berends-Giele → COMIX

Tab. 4 Cross section and evaluation times for different matrix element (phase space) generation methods for multi-gluon scattering at the LHC, given in pb. Numbers were generated on a 2.53 GHz Intel® Core™2 Duo T9400 CPU. For cuts and parameter settings, cf. Tab. 3.

Gleisberg et al., 0808.3672

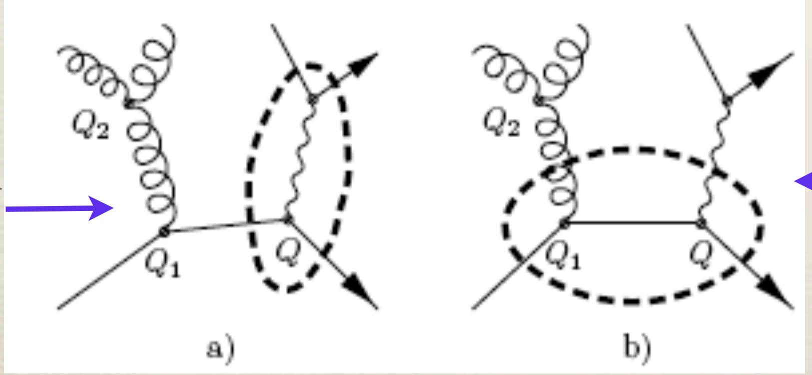
Merging LO with PS

- Want to attach parton shower: describes soft/collinear jets, very high multiplicity allows connections to hadronization
- Don't want to double count emissions from diagrams and PS!

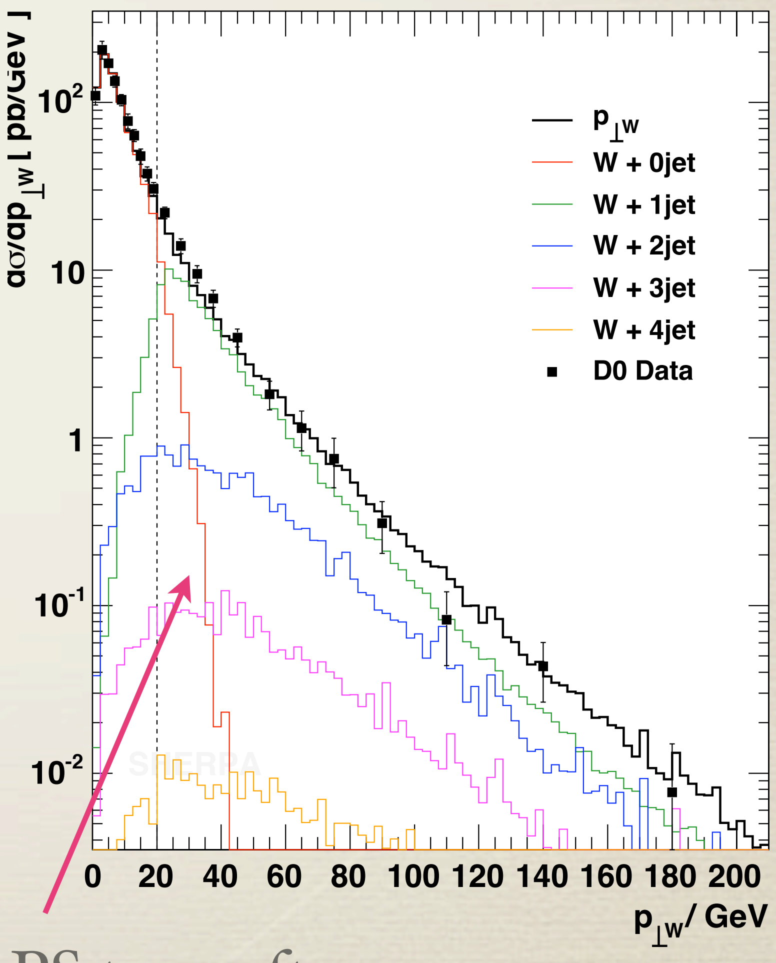
CKKW matching (for W+jets):
(Catani, Krauss, Kuhn, Webber hep-ph/0109231))

- Define jet resolution parameter Q_{cut}
- Select W+n jet process according to $P_n = \frac{\sigma_n}{\sum_i \sigma_i}$
- Generate shower starting from this configuration
- Reweight internal lines with Sudakov factor
- Veto emissions above Q_{cut}

Shower from W+0 jets



Shower from W+1 jet

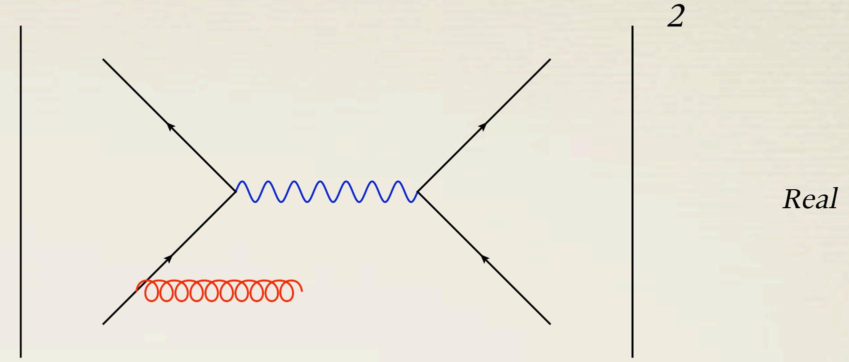


Pure PS too soft

Krauss et al., hep-ph/0409106

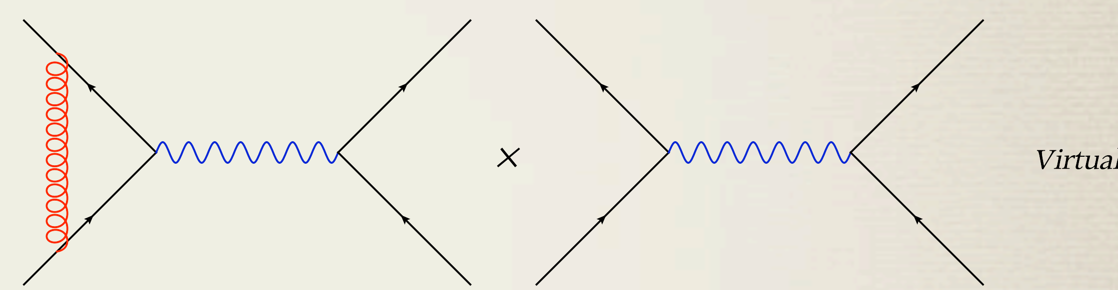
Computing σ: NLO

$$\sigma = \overbrace{\sigma_0}^{LO} + \overbrace{\frac{\alpha_s}{\pi}\sigma_1}^{NLO} + \overbrace{\left(\frac{\alpha_s}{\pi}\right)^2}^{NNLO}\sigma_2 + \ldots$$



Contributions separately singular

- Soft singularities: $E_g \rightarrow 0$
- Collinear singularities: $p_g \,||\, p_i$



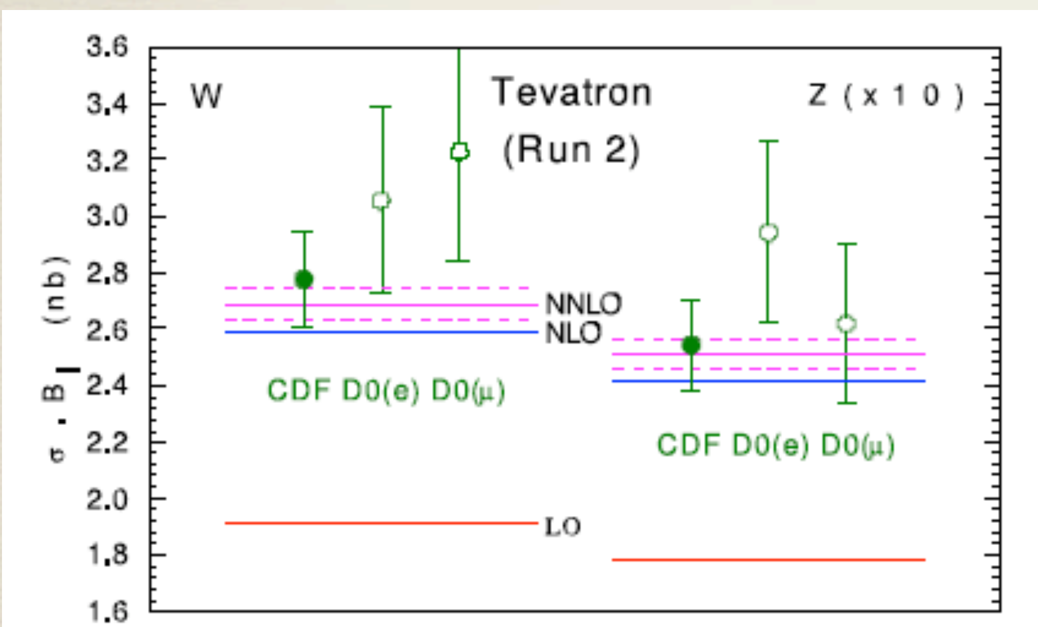
Kinoshita-Lee-Nauenberg (KLN) theorem: singularities cancel after summation over degenerate initial/final states

Cancellation occurs for *infrared-safe* observables: insensitive to soft/collinear radiation

+ Lepton from Z decay η distribution
- Number of partons in event
- p_T=0 for W,Z,H boson (diverges!)

Benefits of NLO

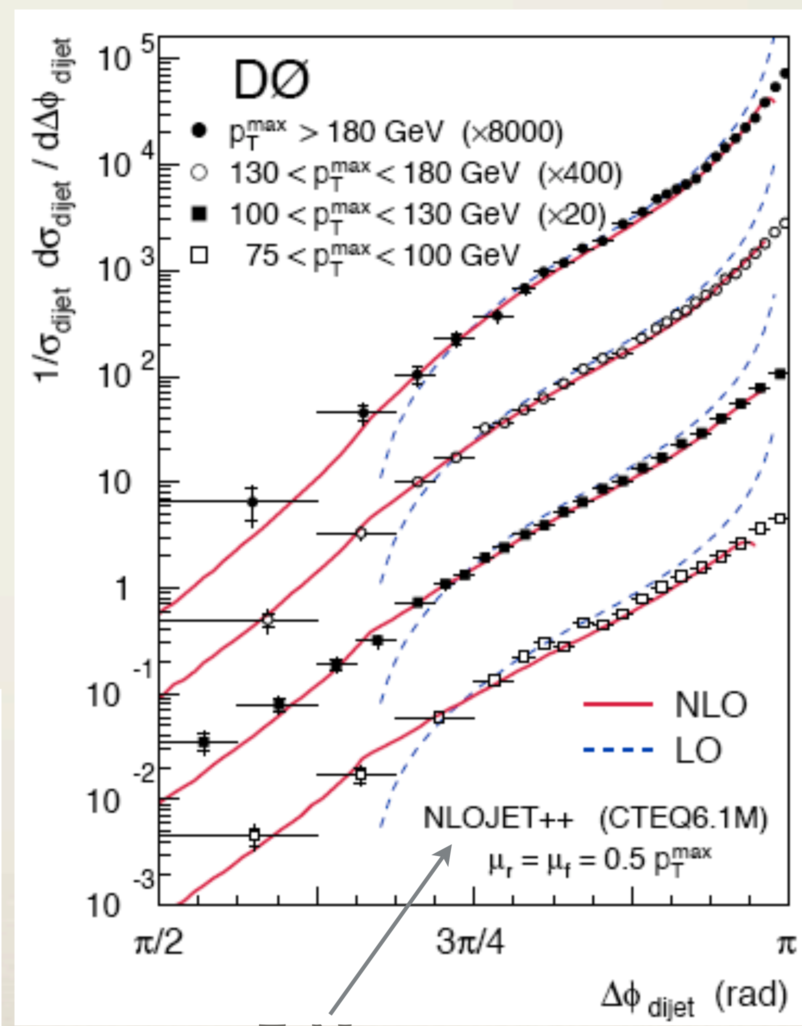
- Improved normalization and smaller residual uncertainty
- Better description of distribution shapes
- First serious quantitative prediction only at NLO



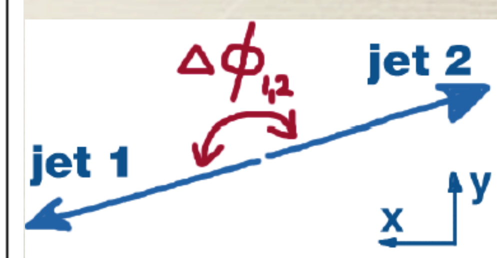
W+jets

number of jets	CDF	LO	NLO
1	53.5 ± 5.6	$41.40(0.02)^{+7.59}_{-5.94}$	$57.83(0.12)^{+4.36}_{-4.00}$
2	6.8 ± 1.1	$6.159(0.004)^{+2.41}_{-1.58}$	$7.62(0.04)^{+0.62}_{-0.86}$
3	0.84 ± 0.24	$0.796(0.001)^{+0.488}_{-0.276}$	$0.882(0.005)^{+0.057}_{-0.138}$

BLACKHAT: Berger et al., 0907.1984



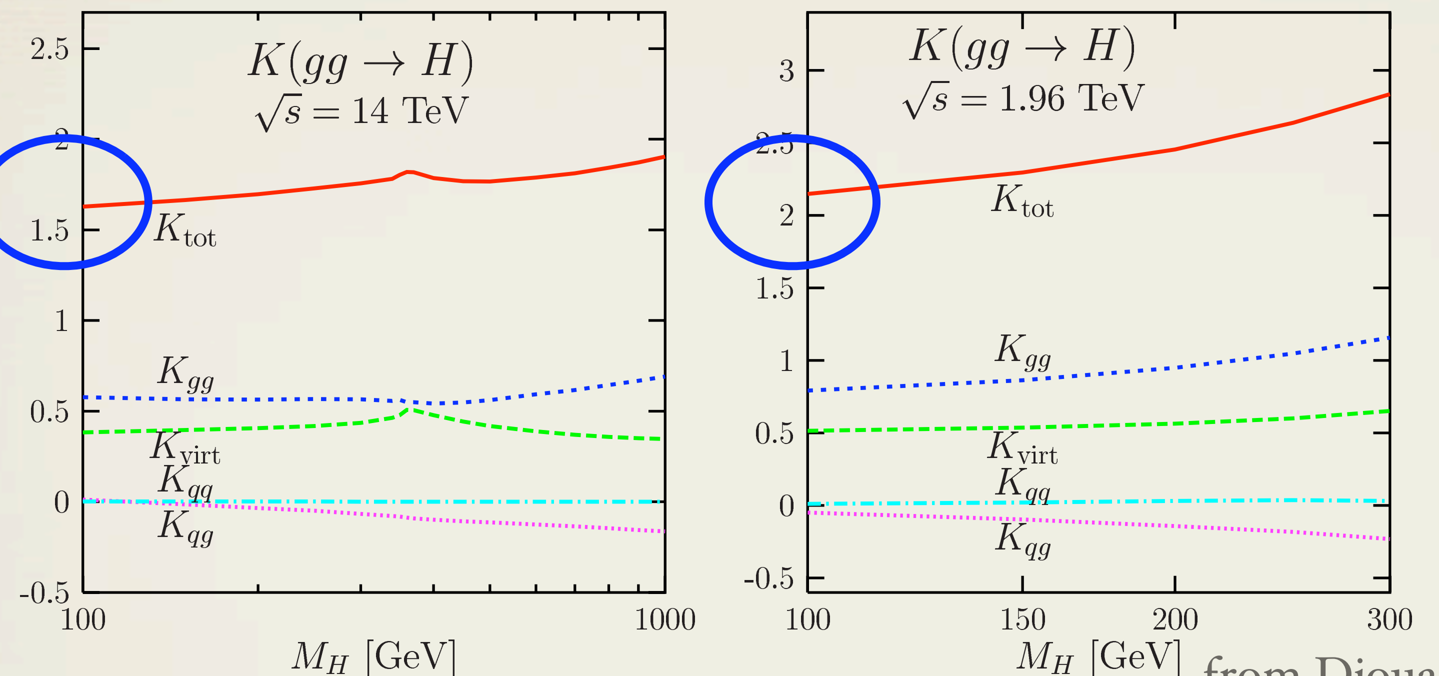
Z. Nagy



Why so large?

- Naive estimate of magnitude: α_s/π~few percent

$K=\sigma_{NLO}/\sigma_{LO}$



quark-initiated: ~30%
gluon-initiated: ~100%

from Djouadi, hep-ph/0503172

- Let's do an example to see what is happening: gg→H (total cross section only)

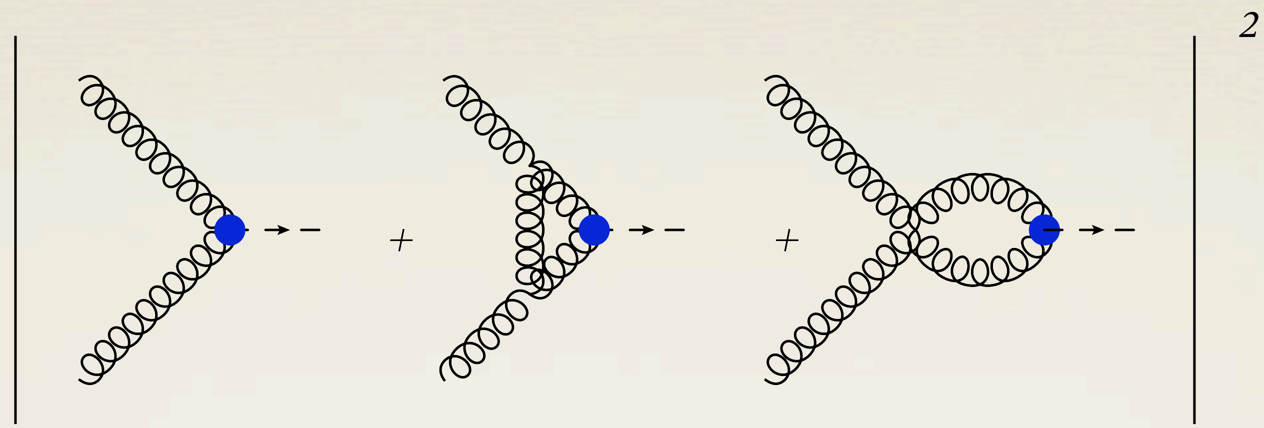
$$\mathcal{L}_{ggH} = -C_1 \frac{H}{v} G^a_{\mu\nu} G^{\mu\nu}_a$$

$$C_1 = -\frac{1}{12}\frac{\alpha_s}{\pi}\left\{1 + \frac{\alpha_s}{\pi}\frac{11}{4} + \ldots\right\}$$

(valid when $M_H<2m_t$)

- Pick a regularization scheme: $d^4k \to d^{4-2\varepsilon}k$
- Calculate real+virtual diagrams
- Renormalize UV and initial-state collinear singularities

Gluon fusion: virtual corrections



$$= \sigma_0 \frac{\alpha_s}{\pi} \mathcal{N}_\epsilon \left(\frac{\mu^2}{\hat{s}} \right)^\epsilon \left\{ -\frac{3}{\epsilon^2} - \frac{3}{\epsilon} - 3 + \frac{11}{2} + \pi^2 \right\} \delta(1 - z)$$

LO as overall normalization

goes to 1 as ε→0

double pole: soft +collinear gluon

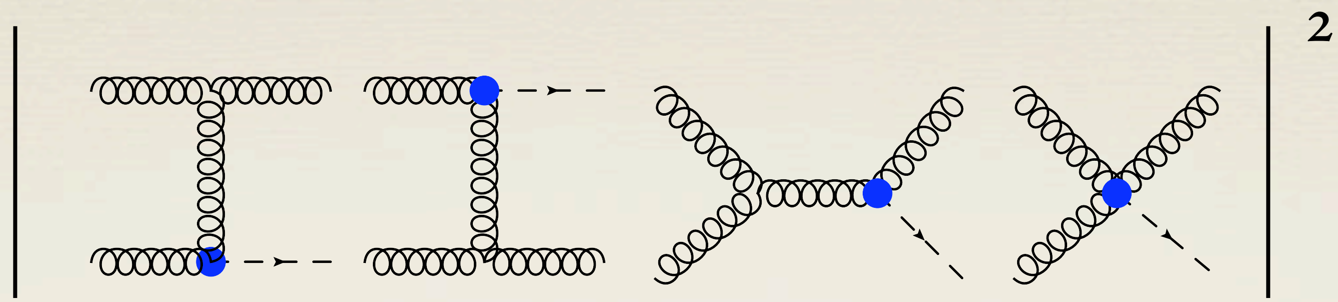
$z = M_H^2/\hat{s} = M_H^2/(x_1 x_2 s)$

all energy into Higgs

11/2: from C_1 term in L_{ggH}

- π^2: $6(-\mu^2/s)^\varepsilon/\varepsilon^2 = (\mu^2/s)^\varepsilon \times (6/\varepsilon^2 + \pi^2 +$imaginary parts$+ \ldots)$
- C_A=**3** comes from emitting gluons from gluons

Gluon fusion: real radiation



Phase space: $d\Pi \quad \sim \quad (1-z)^{1-2\epsilon}\lambda^{-\epsilon}(1-\lambda)^{-\epsilon}$

with $\hat{t} = -\hat{s}(1-z)\lambda,\ \hat{u} = -\hat{s}(1-z)(1-\lambda)$

Matrix elements: $|\bar{M}|^2 \quad \sim \quad \dfrac{1}{\hat{t}\hat{u}} = \dfrac{1}{\hat{s}^2(1-z)^2\lambda(1-\lambda)}$

$$\boxed{d\sigma \quad \sim \quad (1-z)^{-1-2\epsilon}\lambda^{-1-\epsilon}(1-\lambda)^{-1-\epsilon}}$$

singular regulator

λ→0,1: collinear singularities

z→1: soft singularity

Plus distributions: $\lambda^{-1-\epsilon} = -\frac{1}{\epsilon}\delta(\lambda) + \frac{1}{[\lambda]_+} - \epsilon\left[\frac{\ln\lambda}{\lambda}\right]_+ + \mathcal{O}(\epsilon^2) \;\Rightarrow\; \int_0^1 dx\, f(x)[g(x)]_+ = \int_0^1 dx\,[f(x)-f(0)]\,g(x)$

$$\Rightarrow \quad \sigma_0\frac{\alpha_s}{\pi}\mathcal{N}_\epsilon\left(\frac{\mu^2}{\hat{s}}\right)^{\epsilon}\Bigg\{\overbrace{\left[\frac{3}{\epsilon^2}+\frac{3}{\epsilon}+3\right]\delta(1-z)}^{\text{cancels virtual terms}}\overbrace{-\frac{6}{\epsilon}\frac{1}{[1-z]_+}+\frac{6z(z^2-z+2)}{\epsilon}}^{\text{initial-state collinear sings.}}$$

$$+ \quad -\frac{6}{[1-z]_+} + 12\left[\frac{\ln(1-z)}{1-z}\right]_+ - \frac{6(z^2-z+1)^2\ln z}{1-z}$$

$$- \quad 12z(z^2-z+2)\ln(1-z) - \frac{11}{2} + \frac{57z}{2} - \frac{45z^2}{2} + \frac{23z^3}{2}\Bigg\}$$

Gluon fusion: final result

- After renormalization (UV+PDF), arrive at the correction

$$\Delta\sigma = \sigma_0 \frac{\alpha_s}{\pi}\left\{\left(\frac{11}{2}+\pi^2\right)\delta(1-z)+12\left[\frac{\ln(1-z)}{1-z}\right]_+ - 12z(-z+z^2+2)\ln(1-z) - 6\frac{(z^2+1-z)^2}{1-z}\ln(z) - \frac{11}{2}(1-z)^3\right\}$$

$(M^2/s \leq z \leq 1)$

(integration over PDFs ⇒ integration over z)

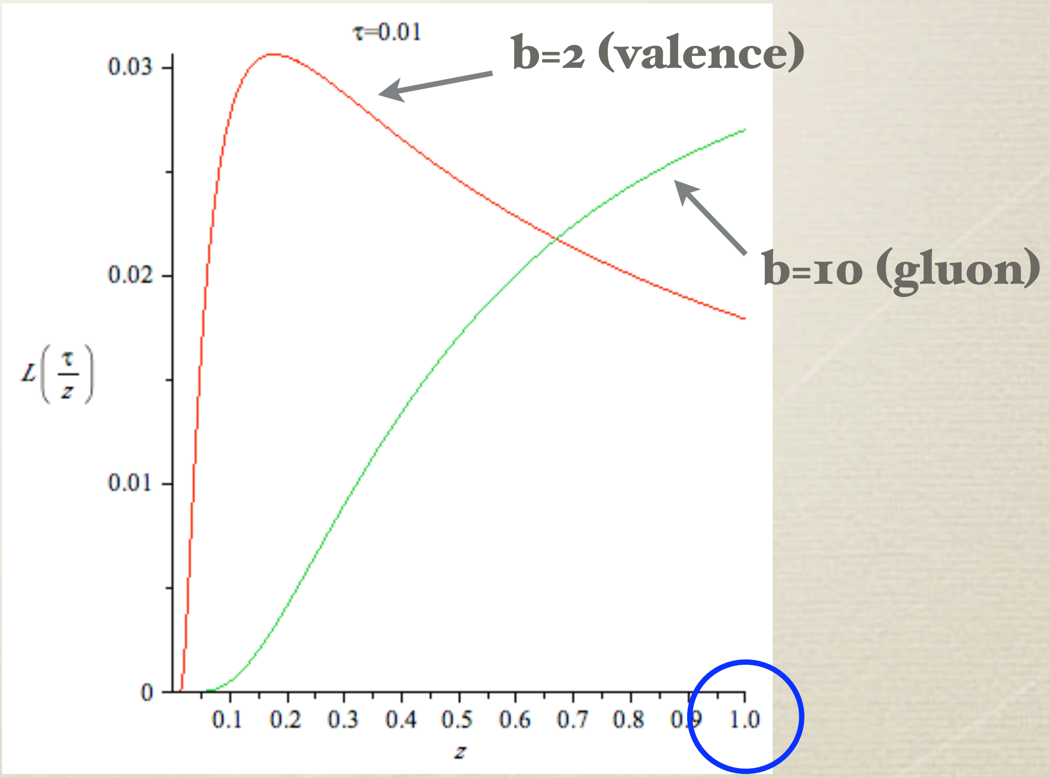
  - First source of large correction: $11/2+\pi^2 \Rightarrow 50\%$ increase
  - Second source: shape of PDFs enhances *threshold* logarithm

$$\sigma_{had} = \tau \int_\tau^1 dz\, \frac{\sigma(z)}{z}\, \mathcal{L}\left(\frac{\tau}{z}\right)$$

$$\mathcal{L}(y) = \int_y^1 dx\, \frac{y}{x}\, f_1(x) f_2(y/x) \quad \text{(partonic luminosity)}$$

Assume $f_i \sim (1-x)^b$; plot L for various b
Look for peak near $z \approx 1$

⇒Sharp fall-off of gluon PDF enhances correction



Available NLO results

- Corrections can be surprisingly large (time-like π^2, phase-space edges) $\Rightarrow$ should have NLO for all processes, what is known?
- Roughly: $2\to2$ easy and known, $2\to3$ challenging (spurious singularities, algebraic complexity) but doable, only two $2\to4$ results known

Partial listing at http://www.cedar.ac.uk/hepcode/

Some examples:

- MCFM (Campbell, Ellis): V+≤2 jets, VH, H+≤1 jet, QQ
- NLOJET++ (Nagy): ≤3 jets
- DIPHOX (Aurenche et al.): γγ, γ+jet
- VBFNLO (Arnold et al.): many vector-boson fusion signals, backgrounds
- …

process	$\sigma_{NLO,NNLO}$ (by)
$gg \to H$ HIGLU MCFM MC@NLO,POWHEG	S.Dawson, NPB 359 (1991), A.Djouadi, M.Spira, P.Zerwas, PLB 264 (1991) C.J.Glosser *et al.*, JHEP (2002); V.Ravindran *et al.*, NPB 634 (2002) D. de Florian *et al.*, PRL 82 (1999) R.Harlander, W.Kilgore, PRL 88 (2002) (NNLO) C.Anastasiou, K.Melnikov, NPB 646 (2002) (NNLO) V.Ravindran *et al.*, NPB 665 (2003) (NNLO) S.Catani *et al.* JHEP 0307 (2003) (NNLL) G.Bozzi *et al.*, PLB 564 (2003), NPB 737 (2006) (NNLL) C.Anastasiou, R.Boughezal, F.Petriello, JHEP (2008) (QCD+EW)
$q\bar{q} \to (W,Z)H$	T.Han, S.Willenbrock, PLB 273 (1991) O.Brien, A.Djouadi, R.Harlander, PLB 579 (2004) (NNLO)
$q\bar{q} \to q\bar{q}H$	T.Han, G.Valencia, S.Willenbrock, PRL 69 (1992) T.Figy, C.Oleari, D.Zeppenfeld, PRD 68 (2003)
$q\bar{q}, gg \to t\bar{t}H$	W.Beenakker *et al.*, PRL 87 (2001), NPB 653 (2003) S.Dawson *et al.*, PRL 87 (2001), PRD 65 (2002), PRD 67,68 (2003)
$q\bar{q}, gg \to b\bar{b}H$	S.Dittmaier, M.Krämer, M.Spira, PRD 70 (2004) S.Dawson *et al.*, PRD 69 (2004), PRL 94 (2005)
$gb(\bar{b}) \to b(\bar{b})H$ MCFM	J.Cambell *et al.*, PRD 67 (2003)
$b\bar{b} \to (b\bar{b})H$ MCFM	D.A.Dicus *et al.* PRD 59 (1999); C.Balasz *et al.*, PRD 60 (1999). R.Harlander, W.Kilgore, PRD 68 (2003) (NNLO)

process	$\sigma_{NLO,NNLO}$ (by)
$W,Z(\to l\nu, ll)$ MCFM MC@NLO,POWHEG ResBos	W.L.van Neerven *et al*, NBP 382 (1992) R.Hamberg, W.L.van Neerven and T.Matsuura, NPB 359 (1991) (NNLO) C.Anastasiou, L.Dixon, K.Melnikov, F.Petriello (NNLO, distrib.) C.Balazs, C.-P. Yuan, PRD 56 (1997) (resummed NLO)
WW, ZZ, WZ AYLEN/EMILIA MCFM MC@NLO,POWHEG	J.Ohnemus *et al.*, PRD 44 (1991); PRD 43 (1991); PRD 50 (1994) B.Mele *et al.*, NPB 357 (1991) S.Frixione *et al.*, NPB 410 (1993); NPB 383 (1992) L.Dixon *et al.*, NPB 531 (1998); PRD 60 (1999) J.Campbell, R.K.Ellis, F.Tramontano, PRD 60 (1999)
VVV VBFNLO	V.Hankele, D.Zeppenfeld, PLB (2007); F.Campanario *et al.* PRD (2008) A.Lazopoulos, K.Melnikov, F.Petriello, PRD 76 (2007) T.Binoth *et al.* JHEP 0806.082 (2008)
$W,Z+\leq 2j$ MCFM	W.Giele, N.Glover, D.Kosower, NPB 403 (1993) J.Campbell *et al*, PRD 65 (2002); PRD 68 (2003)
$W,Z+3j$	C.Berger *et al.* (Blackhat collaboration), arXiv:00902.2760 R.K.Ellis *et al.* JHEP 0901:012, 2009.
$WW+j$	J.Campbell, R.K.Ellis, G.Zanderighi, JHEP 0712:056 (2007) S.Dittamier, S.Kallweit, P.Uwer, PRL 100 (2008)
$W,Z+Q$ MCFM	W.Giele *et al.*, PLB 372 (1996); E.Berger *et al.*, PRD 54 (1996); M.Aivazia *et al*, PRD 50 (1994); J.Collins, PRD 58 (1998); T.Stelzer *et al.*, PRD 56 (1997); J.Campbell, *et al.*, PRD 69 (2004)
$W,Z+Q\bar{Q}$ MCFM	J.Campbell, R.K.Ellis, PRD 62 (2000) ($m_Q \to 0$) F.Maltoni *et al.*, hep-ph/0505014 ($m_Q \to 0$) Febres Cordero *et al.*, PRD 74 (2006), PRD 78 (2008), arXiv:0906.1923.

process	$\sigma_{NLO,NNLO}$ (by)
$Q\bar{Q}$ MCFM MC@NLO,POWHEG	P.Nason, S.Dawson, R.K.Ellis, NPB 303 (1988); NPB 327 (1989) W.Beenakker *et al.*, PRD 40 (1989); NPB 351 (1991) M.Mangano. P.Nason, G.Ridolfi, NPB 373 (1992) R.Bonciani, S.Catani, M.L.Mangano, P.Nason, NPB 529 (1998) (NNL) N.Kidonakis, R.Vogt, Eur. Phys. J. C 33 (2004), C 36 (2004) (≃NNLO) N. Kidonakis, Mod. Phys. Lett. A 19 (2004) (NNNLL+NNLO) A.Banfi, E.Laenen, PRD 71 (2005) and refs. therein (NLL+NLO) W.Bernreuther *et al.*, NPB 690 (2004) (spin correlations) M.Czakon, A.Mitov, S.Moch, PLB 651 (2007), NPB 798 (2008), arXiv:0811.4119 (2-loop NNLO)
$Q\bar{Q}$+j	S.Dittmaier, P.Uwer, S. Weinzierl, PRL 98:262002 (2008)
$t\bar{t}+b\bar{b}$	A.Bredenstein, A.Denner, S.Dittmaier, S.Pozzorini, arXiv:0905.0110
single top MCFM MC@NLO	M.Smith, S.Willenbrock, PRD 54 (1996) G.Bordes, B.van Eijk, NPB 435 (1995) T.Stelzer *et al.*, PRD 56 (1997) B.W.Harris *et al.*, PRD 66 (2002) Z.Sullivan, PRD 70 (2004) J.Campbell, R.K.Ellis, PRD 70 (2004) Q.-H. Cao *et al*, PRD 71 (2005); hep-ph/0504230
$pp(\bar{p}p) \to \leq 3j$ NLOJET++ JETRAD	W.Giele, N.Glover, D.Kosower, NPB 403 (1993) Z.Kunszt and D.Soper, PRD 46 (1992) W.Kilgore and W.Giele, PRD 55 (1997) Z.Nagy, PRL88 (2002), PRD 68 (2003) ($3j$)

from L. Reina

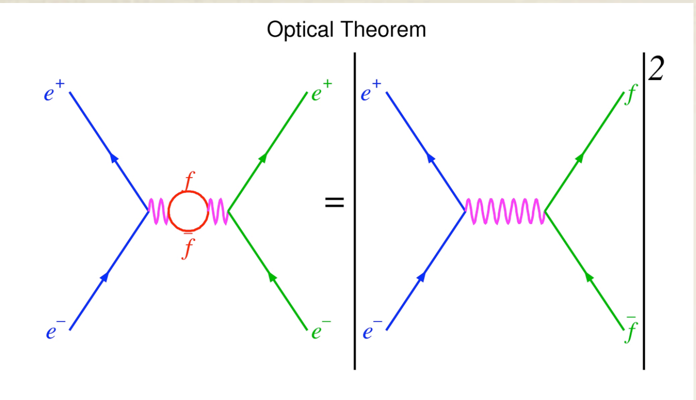
NLO difficulties

- Techniques known to handle real radiation contributions
  Dipole subtraction: construct approximations that reproduce full QCD in singular limits, are analytically integrable (dipoles); cancel poles, numerically integrate full QCD - dipoles (Catani, Seymour hep-ph/9605323)

- Hard part are the loops for 2→3 and beyond...

Factorial growth of diagrams and enormous algebraic expressions, final results often simpler then intermediate steps ⇒ better organizing principle?

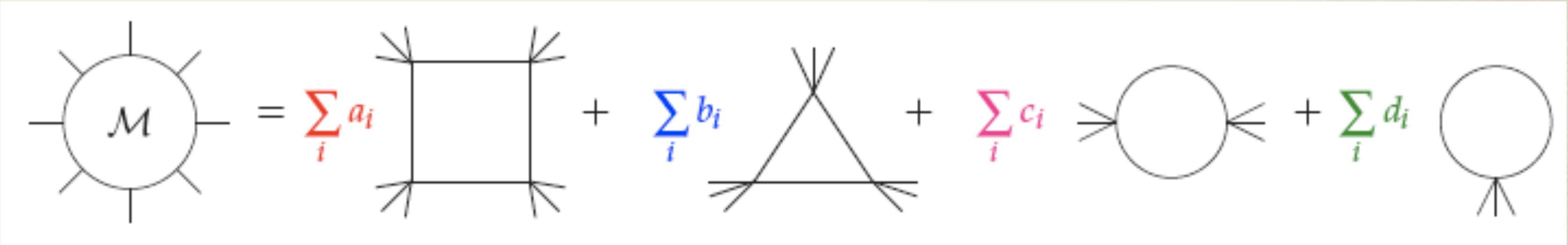
Unitarity and NLO amplitudes



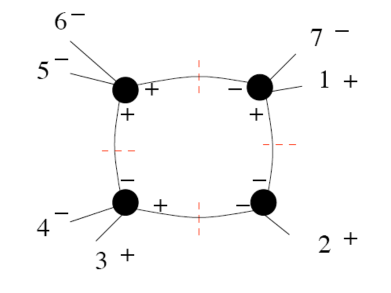
Put loop propagators on-shell (“cut” them) to get imaginary parts from trees

Some success using this+singular limits to construct loops from trees for multi-leg processes Bern, Dixon, Dunbar, Kosower, 1990s

Can decompose 1-loop amplitudes into basis of scalar integrals:



Try to isolate box coefficients a_i by cutting 4 propagators
Only find a solution for *complex* momenta Britto, Cachazo, Feng 2004



$$\text{coeff} = \frac{1}{2}\frac{[\ell_1\,\ell_4]^3}{[\ell_1\,2][2\,\ell_4]}\frac{[4\,\ell_2]^3}{[\ell_2\,\ell_1][\ell_1\,3][3\,4]}\frac{[5\,6]^3}{[6\,\ell_3][\ell_3\,\ell_2][\ell_2\,5]}\frac{[\ell_3\,7]^3}{[7\,1][1\,\ell_4][\ell_4\,\ell_3]}$$

$$= -\frac{\langle 1\,2\rangle^3\langle 2\,3\rangle^3[5\,6]^3}{\langle 7\,1\rangle\langle 3\,4\rangle\langle 2|P_{3,4}|5]\langle 2|P_{7,1}|6]\langle 2|P_{3,4}P_{5,6}|7\rangle\langle 2|P_{7,1}P_{5,6}|4\rangle}$$

$$\frac{1}{2}\sum_{\mathcal{S}} A_1^{\text{tree}} A_2^{\text{tree}} A_3^{\text{tree}} A_4^{\text{tree}}$$

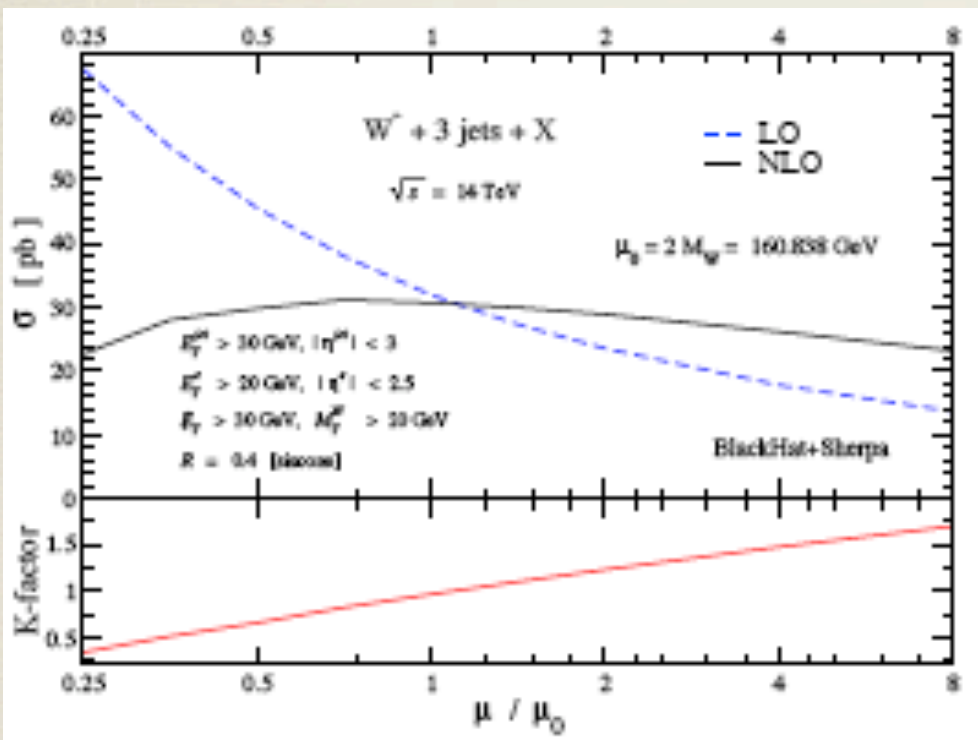
No 1-loop diagrams! Just compute tree graphs... and we know recursive techniques, can do numerically

Unitarity and NLO amplitudes

- Recipe for using unitarity to construct an amplitude:
  - 4-particle cuts to get boxes, 3 -particle cuts to get triangles (subtract 3-particle cut of boxes), ...; scalar integral coefficients are tree-level amplitudes that can be efficiently computed and evaluated numerically
- In d=4, 1-loop amplitudes are "cut-constructible"; in $d=4-2\varepsilon$, terms of the form $1/\varepsilon \times \varepsilon$ aren't obtainable from cuts
  - Special Feynman rules/tree-like recursion to get these "rational" terms (Ossola, Papadopoulos, Pittau 0802.1876; Berger, Bern, Dixon, Forde Kosower hep-ph/0604195)
  - Compute in multiple integer d (d=5,6 for example) and use known polynomial dependence to reconstruct $d=4-2\varepsilon$ (Giele, Kunszt, Melnikov 0801.2237)
- Three primary groups:
  - Blackhat (Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre)
  - CutTools (Ossola, Papadopoulos, Pittau)
  - Rocket (Ellis, Kunszt, Melnikov, Zanderighi)

2→4 at NLO

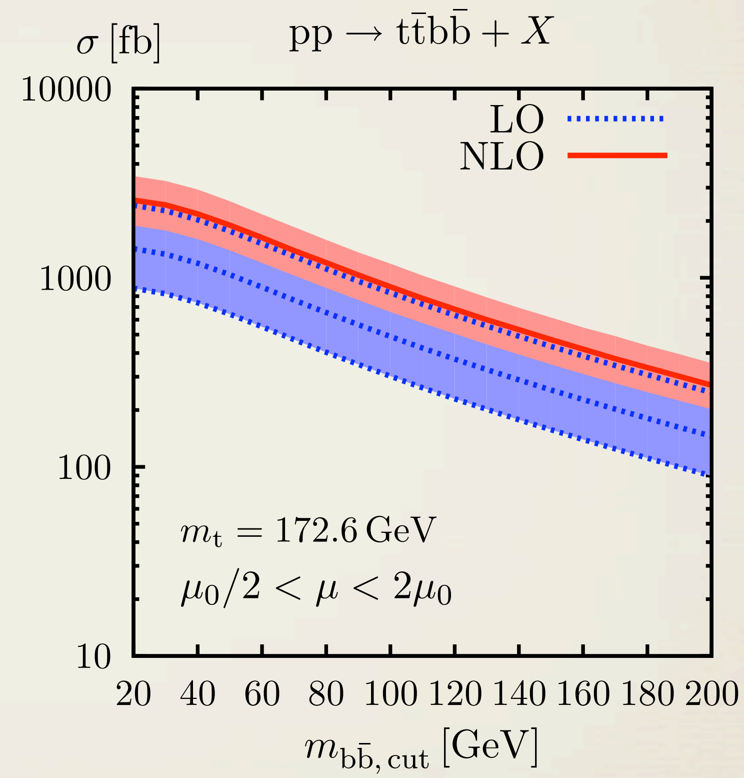
W+3 jets



Unitarity-based approach

Large NC: Rocket 0906.1445
Full QCD: Blackhat 0907.1984

ttbb: background to ttH, important for bottom Yukawa measurement



Traditional Feynman diagrams

Bredenstein, Denner, Dittmaier, Pozzorini 0905.0110

Lots of activity in this area!

Merging NLO with PS

- Want to combine NLO with parton shower ⇒ first hard emission described by NLO calculation, loops give right normalization
- Need to avoid double counting real-emission corrections
- Two working programs: MC@NLO (Frixione, Webber), POWHEG (Frixione, Nason, Oleari)

$$d\sigma_{\mathrm{POWHEG}} = \overline{B}(\mathbf{\Phi}_n)\, d\mathbf{\Phi}_n \left\{ \Delta(\mathbf{\Phi}_n, p_T^{\mathrm{min}}) + \frac{R(\mathbf{\Phi}_n, \Phi_r)}{B(\mathbf{\Phi}_n)} \Delta(\mathbf{\Phi}_n, p_T)\, d\Phi_r \right\}$$

$$\overline{B}(\mathbf{\Phi}_n) = B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n) + \int d\Phi_r \left[R(\mathbf{\Phi}_n, \Phi_r) - C(\mathbf{\Phi}_n, \Phi_r) \right]$$

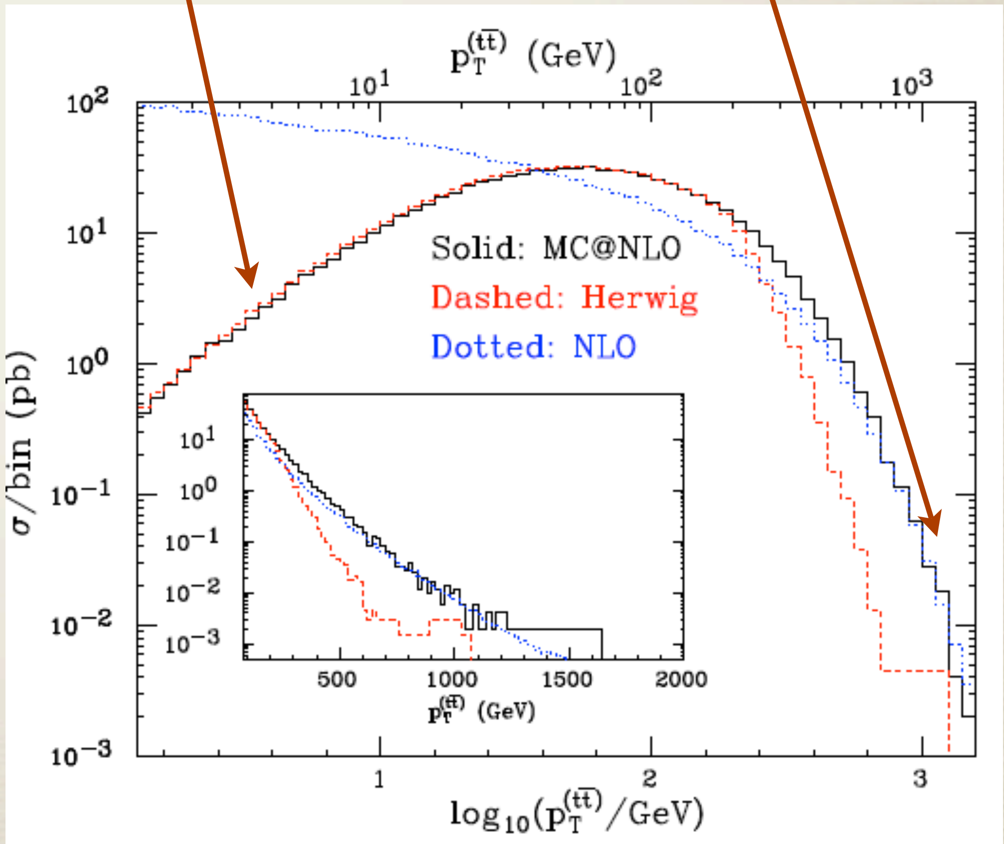
$$\Delta(\mathbf{\Phi}_n, p_T) = \exp\left[-\int d\Phi_r' \frac{R(\mathbf{\Phi}_n, \Phi_r')}{B(\mathbf{\Phi}_n)} \theta\left(k_T(\mathbf{\Phi}_n, \Phi_r') - p_T\right) \right]$$

Virtual corrections included together with counterterms

full real radiation in modifed Sudakov factor

Correct at low p_T

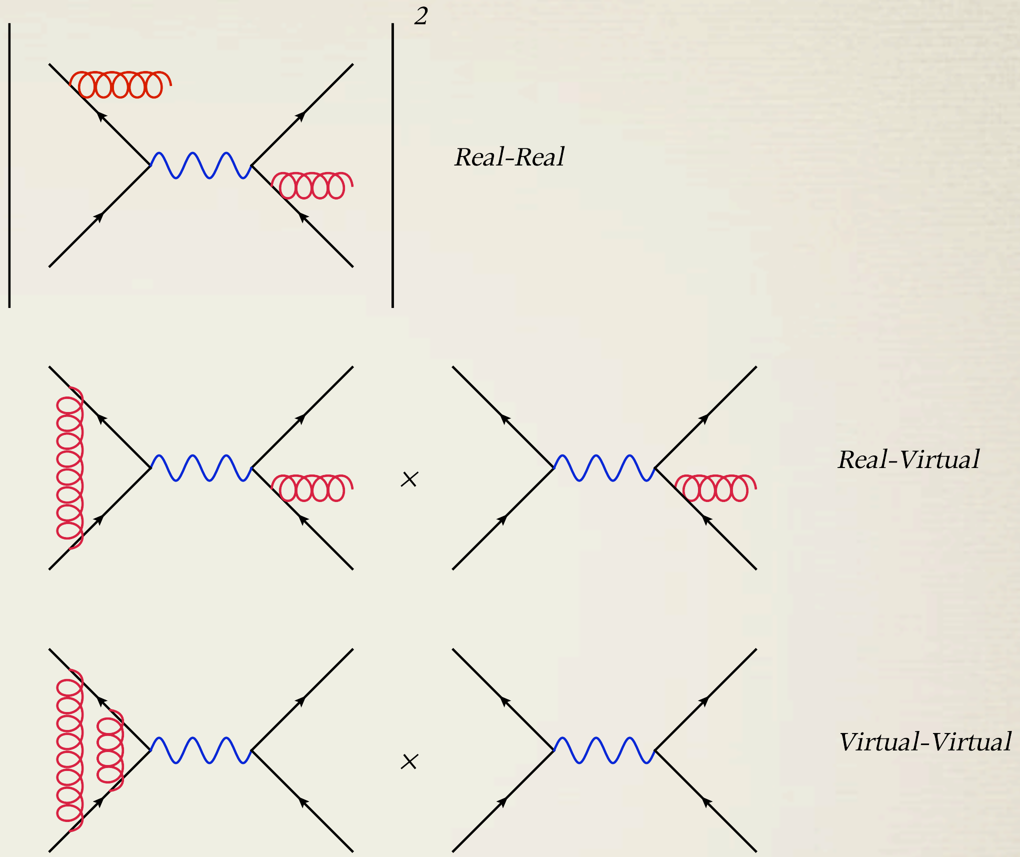
Matches to NLO art high p_T



Correct normalization to $O(\alpha_s)$, matches to NLO hard emission at high p_T, and shower at low p_T

Computing σ: NNLO

$$\sigma = \overbrace{\sigma_0}^{LO} + \overbrace{\frac{\alpha_s}{\pi}\sigma_1}^{NLO} + \overbrace{\left(\frac{\alpha_s}{\pi}\right)^2 \sigma_2}^{NNLO} + \ldots$$



When is NNLO necessary?

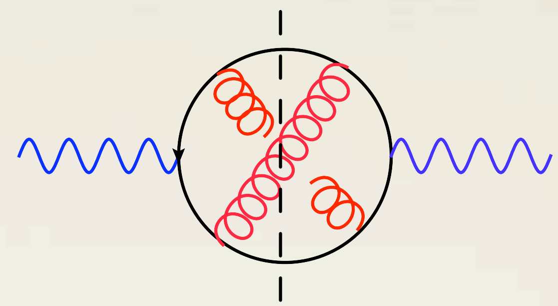
- ☑ When NLO corrections are large, and NNLO is needed to check expansion (gg→H)
- ☑ For benchmark processes where high precision is needed (DIS, Drell-Yan for PDFs, $e^+e^-\to 3$ jets for α_s)

- Organize by # of scales that appear in result
  - no-scale (only Q^2, determined by dimensional analysis): $e^+e^-\to$hadrons
  - 1-scale, inclusive hadron-collider cross sections: pp→H,W,Z (M^2/s)
  - 2-scale, single differential distributions: $d\sigma/dM/dY$
  - all-scales: completely differential results

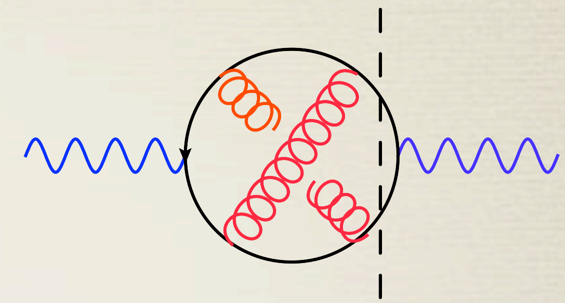
0-scale problems

- Use optical theorem, map to the calculation of loop integrals

σ($\gamma^*\to$hadrons)=Im($\gamma^*\to\gamma^*$)/s

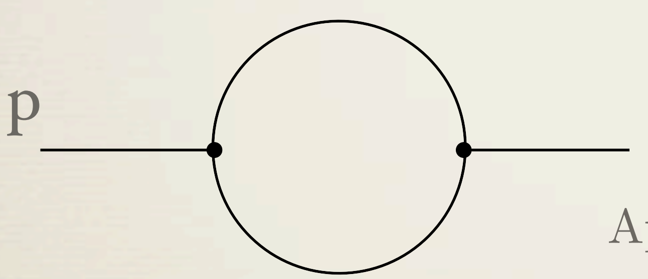


Real-Real cut



Virtual-Virtual cut

- Integration-by-parts to reduce loops integrals to a few “master integrals” Chetyrkin, Tkachov 1981



Set $\int d^d k \, \frac{\partial}{\partial k^\mu}\left[\frac{k^\mu}{k^{2\nu_1}(k+p)^{2\nu_2}}\right] = 0$

Derive $(d-2\nu_1-\nu_2)\mathcal{I}(\nu_1,\nu_2) - \nu_2\mathcal{I}(\nu_1-1,\nu_2+1) + \nu_2 p^2 \mathcal{I}(\nu_1,\nu_2+1) = 0$

Apply to $\mathcal{I}(1,1) \Rightarrow \mathcal{I}(1,2) = -\frac{d-3}{p^2}\mathcal{I}(1,1)$

$$\mathcal{I}(\nu_1,\nu_2) = \int d^d k \, \frac{1}{k^{2\nu_1}(k+p)^{2\nu_2}}$$

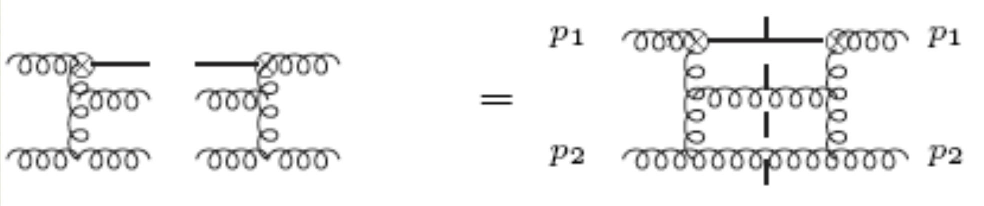
$\Rightarrow$ algebraically relate different integrals

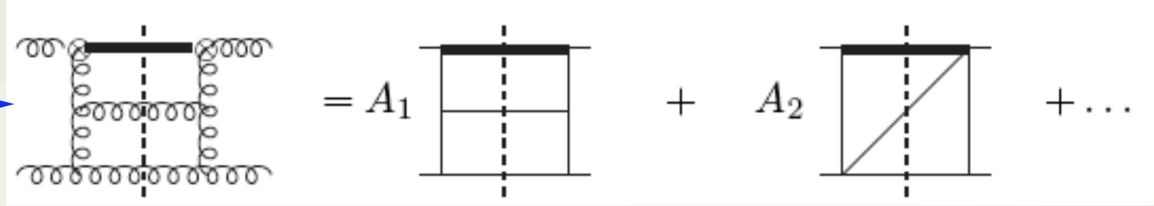
$$\begin{aligned} R^{\overline{\mathrm{MS}}}(s) = 3\sum_f Q_f^2 \Big(1 &+ \bar{\alpha}_s/\pi + (\bar{\alpha}_s/\pi)^2\{+\tfrac{365}{24} - 11\zeta(3) - N_f[\tfrac{11}{12} - \tfrac{2}{3}\zeta(3)]\} \\ &+ (\bar{\alpha}_s/\pi)^3\{+\tfrac{87029}{288} - \tfrac{1103}{4}\zeta(3) + \tfrac{275}{6}\zeta(5) + N_f[-\tfrac{7847}{216} + \tfrac{262}{9}\zeta(3) - \tfrac{25}{9}\zeta(5)] \\ &+ N_f^2[+\tfrac{151}{162} - \tfrac{19}{27}\zeta(3)] - \pi^2/48(11 - \tfrac{2}{3}N_f)^2\} + O(\alpha_s^4)\Big) \\ &+ \Big(\sum_f Q_f\Big)^2 (\bar{\alpha}_s/\pi)^3[\tfrac{55}{72} - \tfrac{5}{3}\zeta(3)] + O(\alpha_s^4)\,. \end{aligned}$$

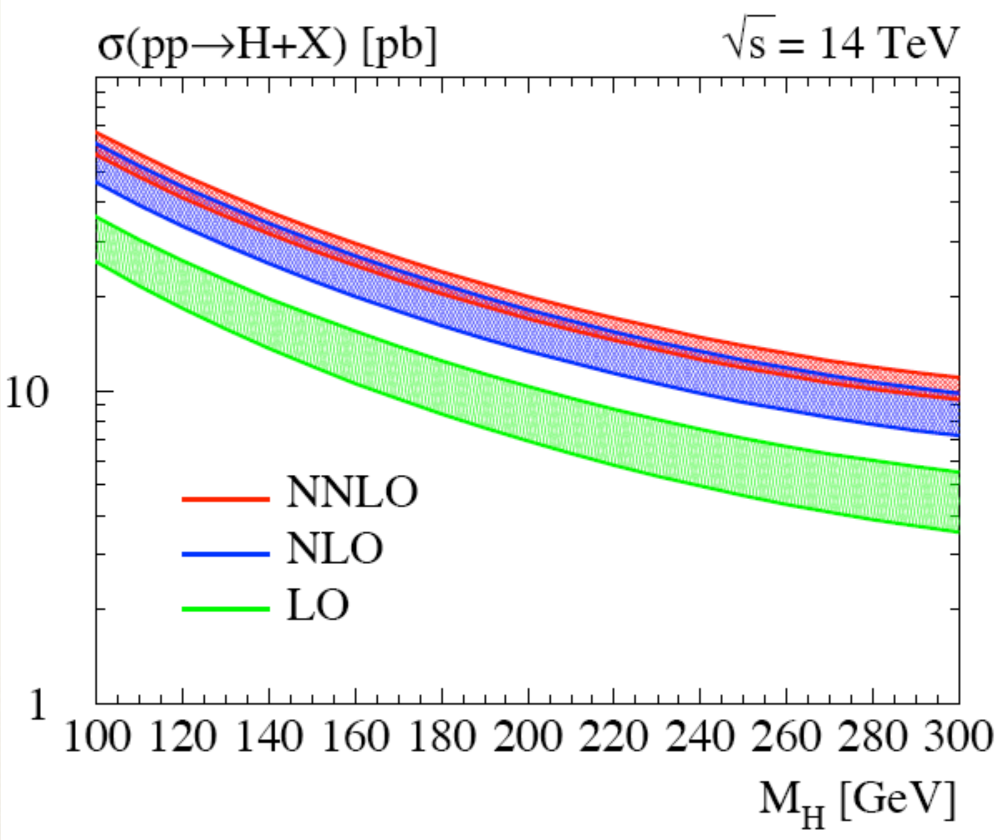
Gorishny, Kataev, Larin 1988; Surguladze, Samuel 1991

1-scale problems

- Same IBP technology can be applied to hadron collider cross sections (Anastasiou, Melnikov hep-ph/0207004) $\Rightarrow$ first applied to Higgs







Perturbative expansion under control

Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven 2002-3

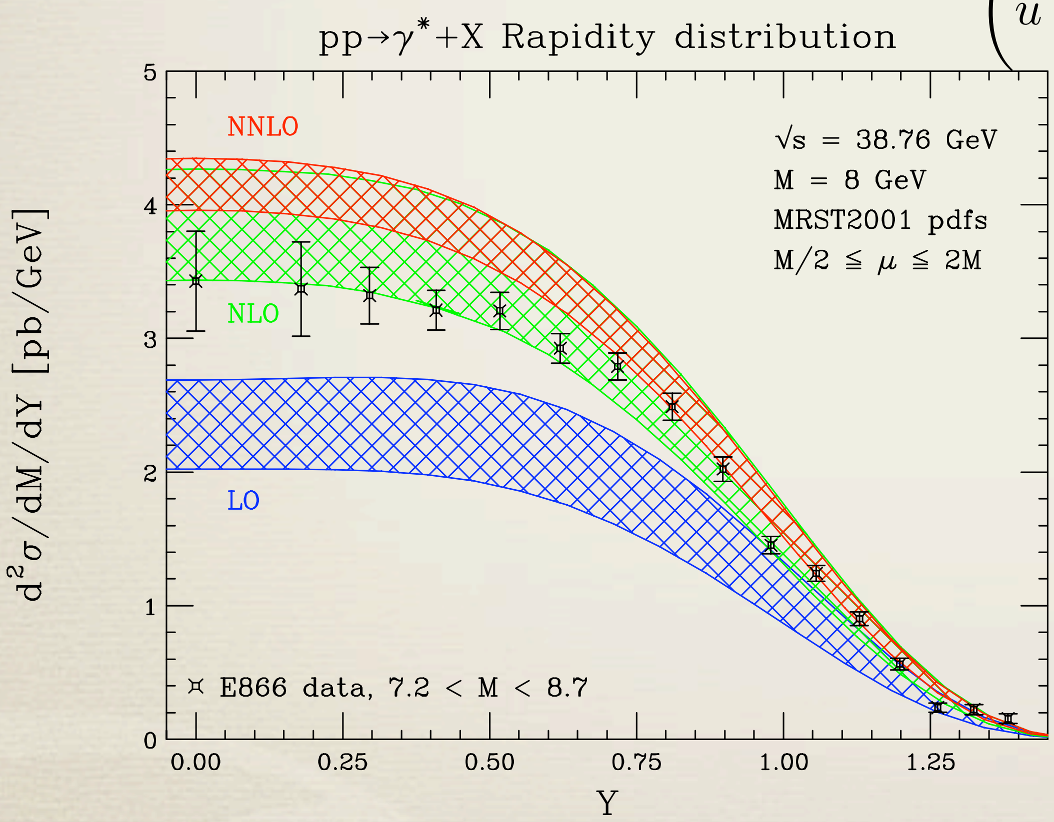
2-scale problems

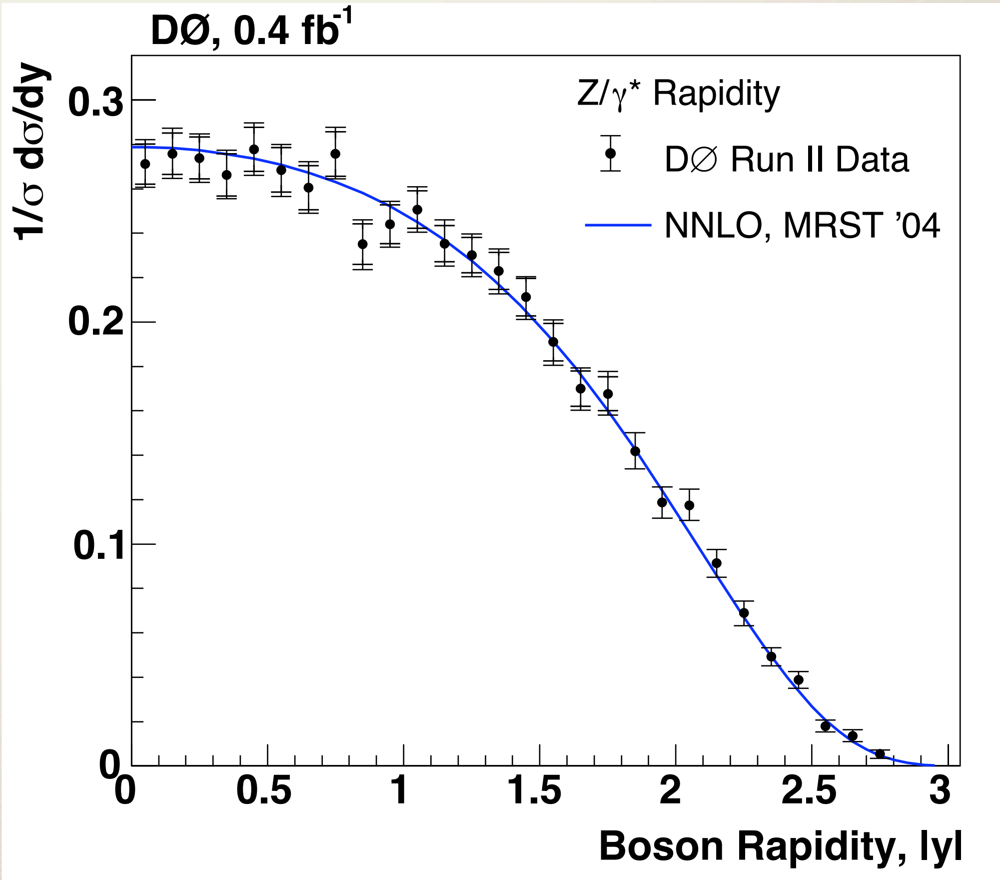
- W, Z rapidity distributions: depend on M^2/s and Y ⇒ introduce a fictitious particle to allow use of IBP with rapidity constraint

Anastasiou, Dixon, Melnikov, FP 2003

$$\overbrace{\delta\left(\frac{p_V\cdot p_1}{p_V\cdot p_2}-u\right)}^{\text{phase-space constraint}} \to \overbrace{\frac{p_V\cdot p_2}{p_V\cdot(p_1-up_2)-i0}}^{\text{fictitious propagator}} - \text{c.c.}$$

$$\left(u=\frac{x_1}{x_2}e^{-2Y}\right)$$





Important constraint on PDFs from fixed-target scattering (high-x quarks)

Fully differential NNLO

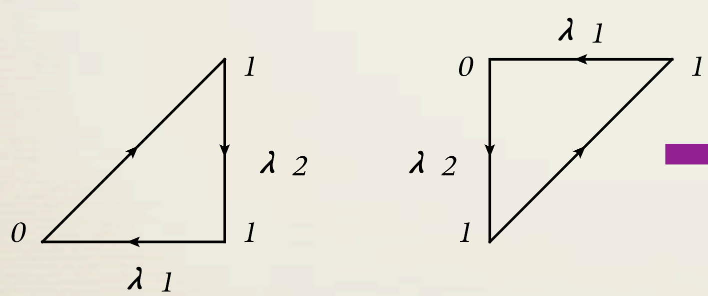
- Desirable to account fully for experimental constraints
- How to arrange singularity cancellation between real and virtual graphs for numerical integration?

Utilize regulators in explicit phase-space parametrizations

$$d\Pi_E = N \int_0^1 \mathrm{d}\lambda_1 \mathrm{d}\lambda_2 \mathrm{d}\lambda_3 \mathrm{d}\lambda_4 [\lambda_1(1-\lambda_1)]^{1-2\epsilon}[\lambda_2(1-\lambda_2)]^{-\epsilon}[\lambda_3(1-\lambda_3)]^{-\epsilon} \times [\lambda_4(1-\lambda_4)]^{-\epsilon-1/2} D^{2-d},$$

"Entangled" singularities:
$$\mathcal{I} = \int_0^1 dx\, dy \, \frac{\lambda_1^\epsilon \lambda_2^\epsilon}{(\lambda_1+\lambda_2)^2}$$

Anastasiou, Melnikov, FP 2003-2004 for Higgs, W, Z



$$\mathcal{I} = \int_0^1 dx\, dy \, \frac{\lambda_1^{-1+2\epsilon}\lambda_2^\epsilon}{(1+\lambda_2)^2} + \int_0^1 dx\, dy \, \frac{\lambda_2^{-1+2\epsilon}\lambda_1^\epsilon}{(1+\lambda_1)^2}$$

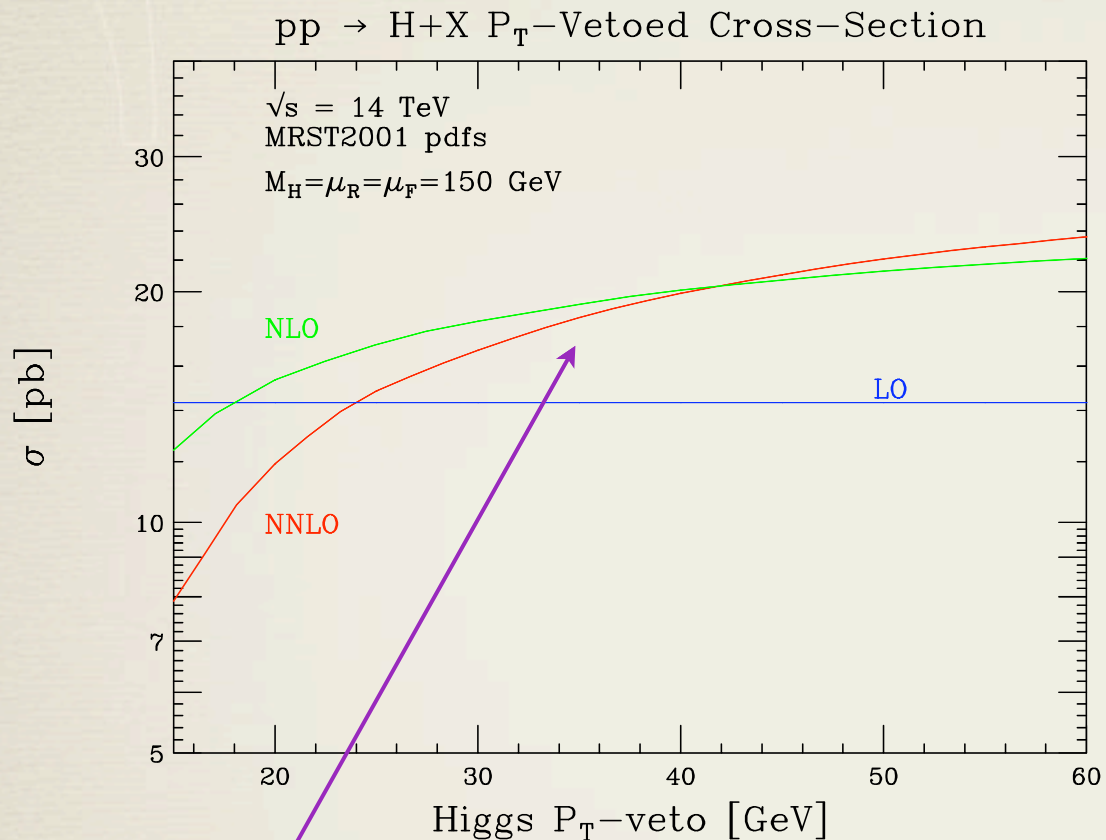
Use singular structure of QCD to build analytically-integrable subtraction terms

$$\begin{aligned}\mathrm{d}\sigma_{NNLO} = & \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^R_{NNLO} - \mathrm{d}\sigma^S_{NNLO}\right) \\ & + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) \\ & + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^S_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^{V,2}_{NNLO}\end{aligned}$$

Gehrmann, Gehrmann de-Ridder, Glover 2004-2007 for e⁺e⁻→3 jets; Catani, Grazzini 2007 for Higgs; many others

Phenomenology at NNLO

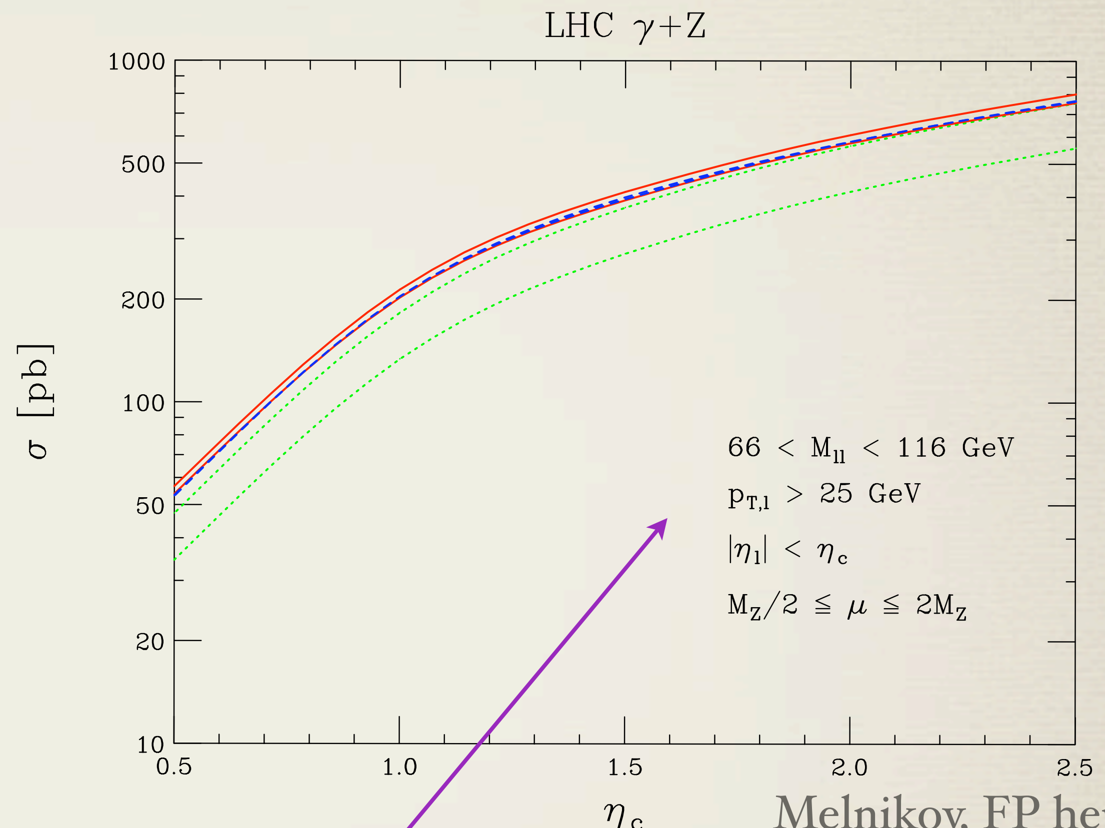
Higgs at LHC:



Anastasiou, Melnikov, FP hep-ph/0501130

NNLO corrections have kinematic dependence! Not just constant reweighting of PYTHIA

W,Z at LHC:



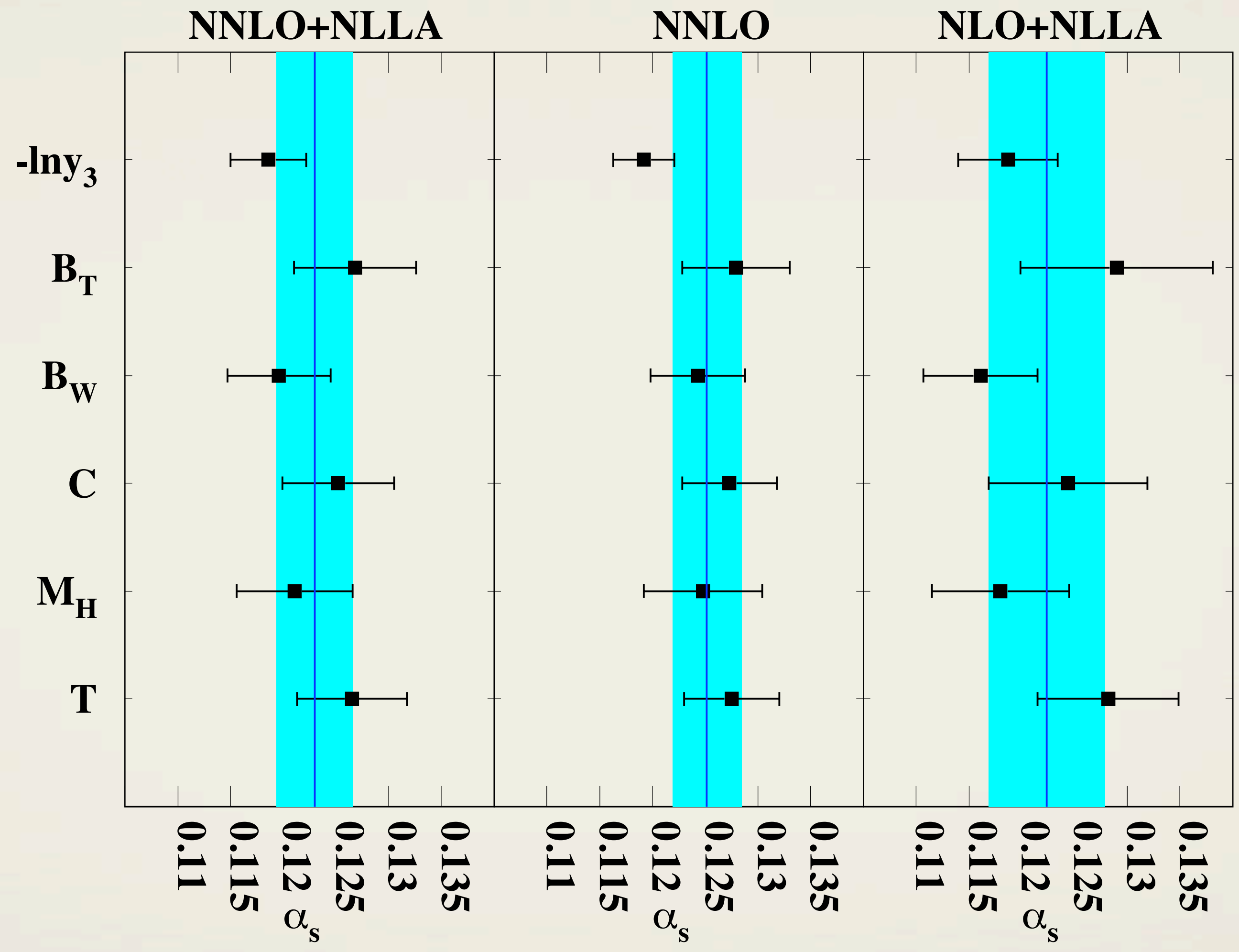
Melnikov, FP hep-ph/0609070

Include acceptance cuts, spin correlations for percent-level "partonic-luminosity monitor" at LHC ⇒ normalize other cross sections to this, small experimental and theory errors

Dittmar, Pauss, Zurcher hep-ex/9705004

Phenomenology at NNLO

- **$e^+e^-\to 3$ jets:** Extract α_s from LEP event shapes



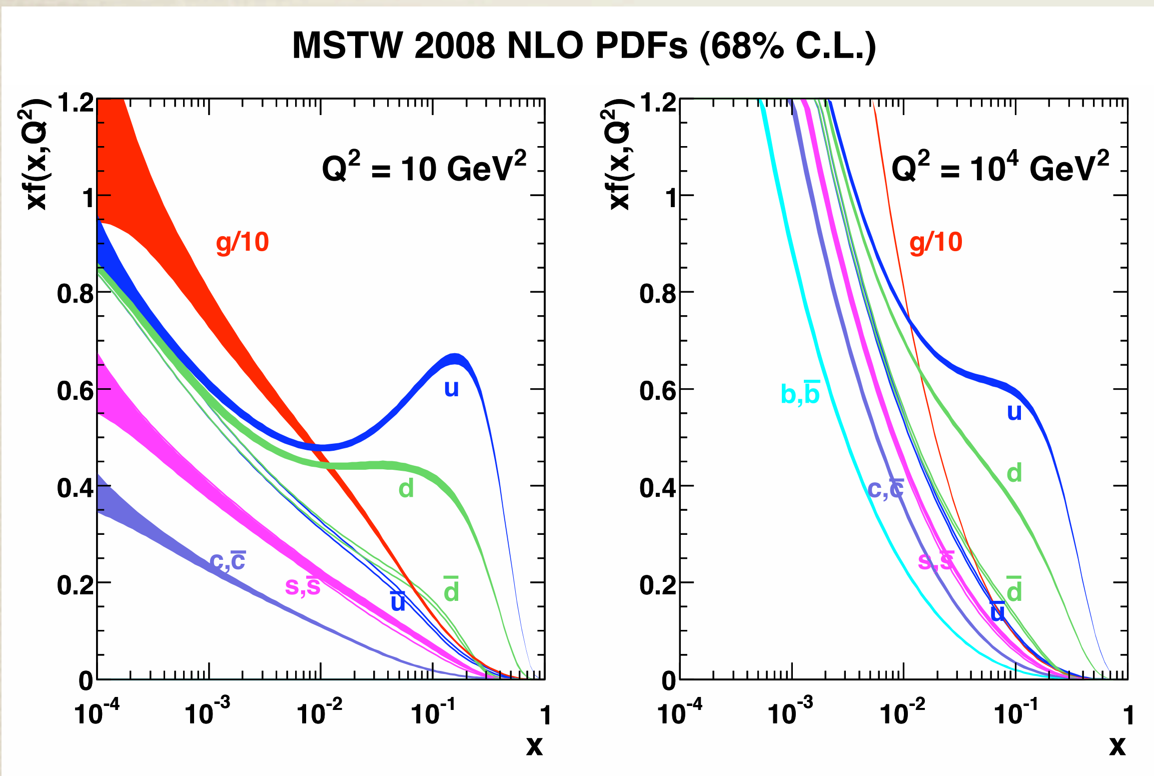
theory still the largest error

$$\alpha_s(M_Z) = 0.1224 \pm 0.0009(\mathrm{stat}) \pm 0.0009(\mathrm{exp}) \pm 0.0012(\mathrm{had}) \pm 0.0035(\mathrm{theo})$$

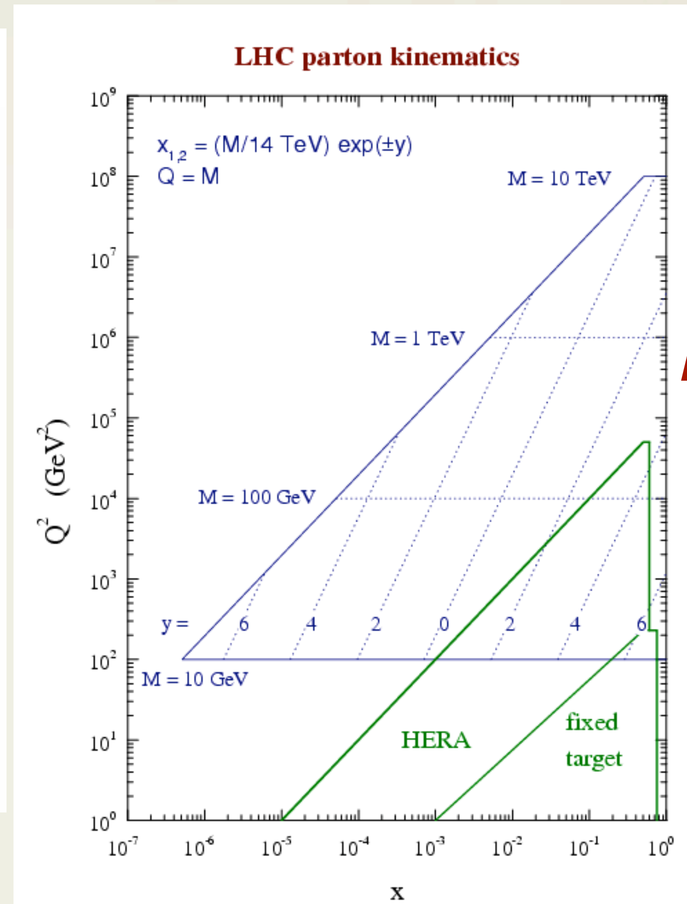
Dissertori, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, Luisoni, Stenzel 0711.4711, 0712.0327, 0906.3436; Weinzierl 0807.3241; Becher, Schwartz 0803.0342

PDFs

- Enter every hadron collider prediction; must be understood!



Lots of gluons!



Q^2 evolution perturbative (NNLO DGLAP kernels: Moch, Vermaseren, Vogt 2004)

Fits by CTEQ, MSTW, Alekhin, NNPDF

DIS, fixed-target DY, Tevatron jets+W,Z

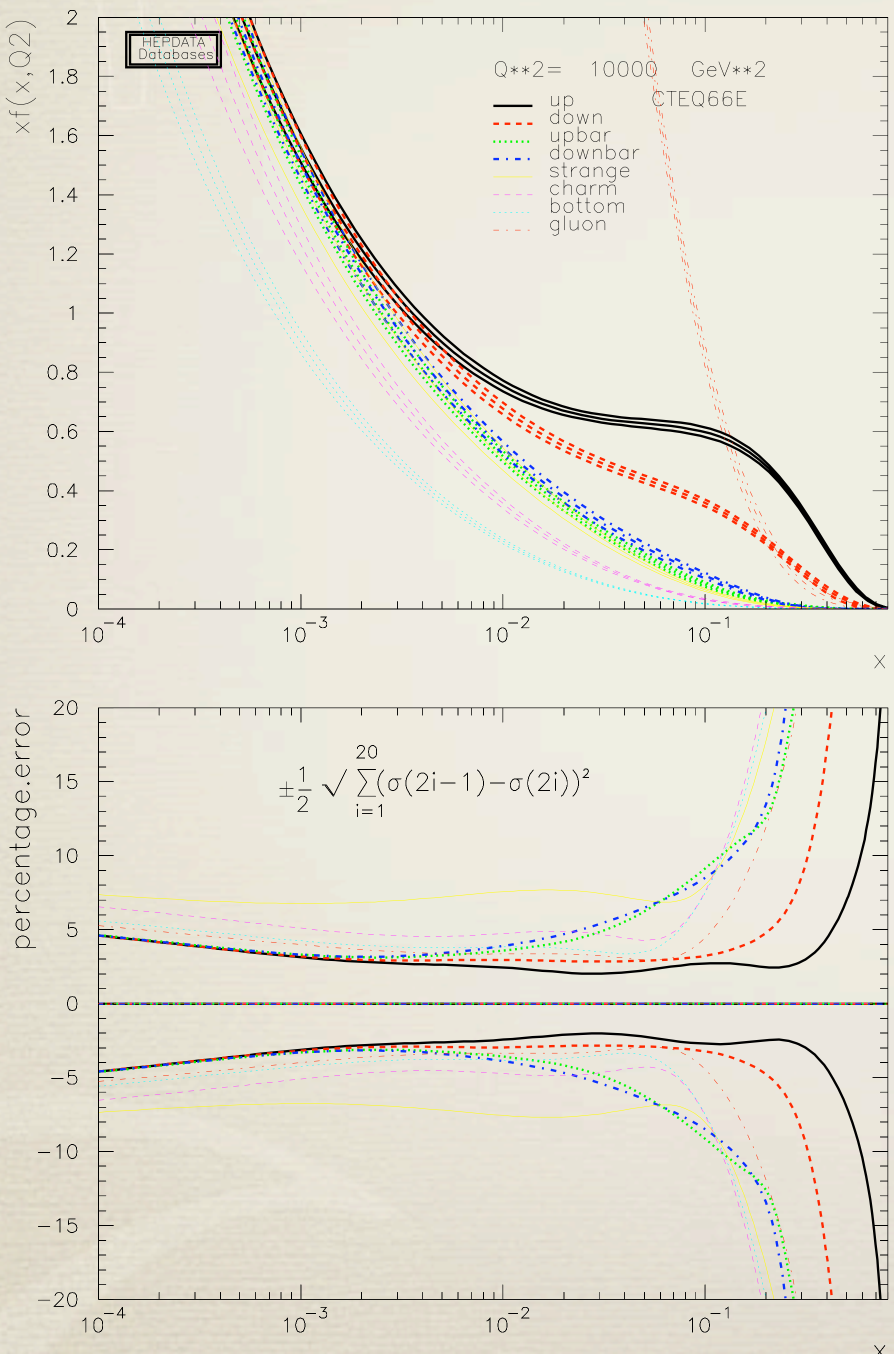
Only known at NLO

Process	Subprocess	Partons	x range
$\ell^\pm \{p,n\} \to \ell^\pm X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$\ell^\pm n/p \to \ell^\pm X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
$pp \to \mu^+\mu^- X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+\mu^- X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu})\, N \to \mu^-(\mu^+)\, X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu\, N \to \mu^-\mu^+ X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu}\, N \to \mu^+\mu^- X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^\pm p \to e^\pm X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
$e^+ p \to \bar{\nu} X$	$W^+ \{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
$e^\pm p \to e^\pm c\bar{c}\, X$	$\gamma^* c \to c,\ \gamma^* g \to c\bar{c}$	$c,\ g$	$0.0001 \lesssim x \lesssim 0.01$
$e^\pm p \to \mathrm{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \to \mathrm{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^\pm \to \ell^\pm \nu)\, X$	$ud \to W, \bar{u}\bar{d} \to W$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
$p\bar{p} \to (Z \to \ell^+\ell^-)\, X$	$uu, dd \to Z$	d	$x \gtrsim 0.05$

TeV HERA Fixed target

PDF errors

- Published sets come with errors... what do they mean?



CTEQ 6.6, http://durpdg.dur.ac.uk/

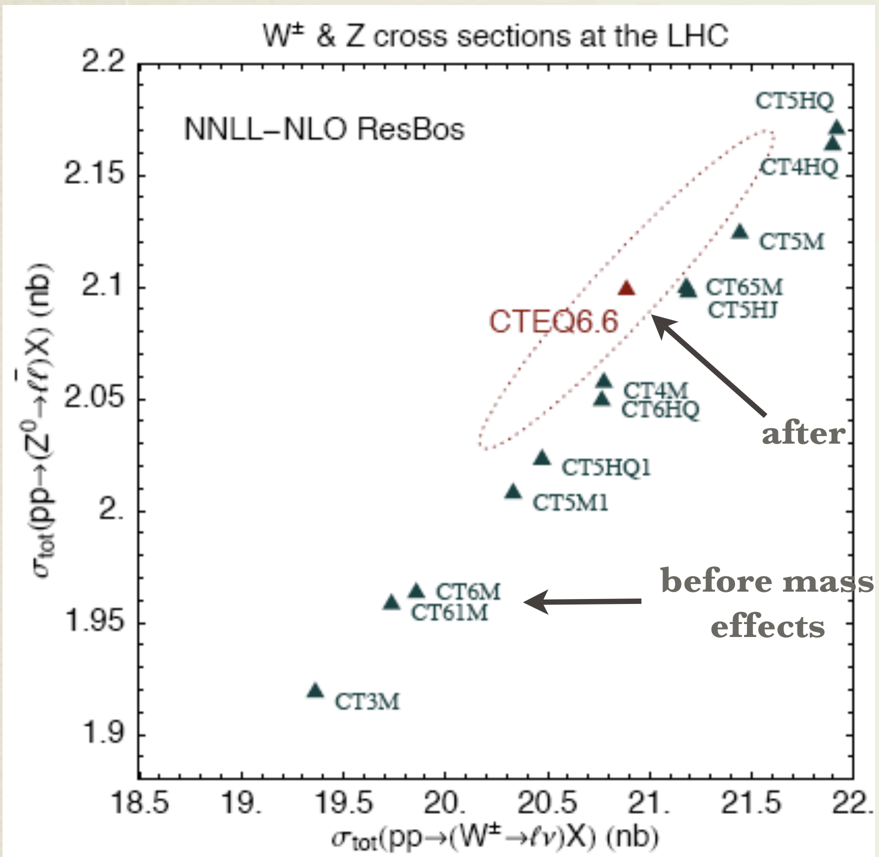
- There are many sources of uncertainty in the PDFs, some of which we've touched on
  - Data set choice
  - Kinematic cuts
  - Parametrization choices
  - Treatment of heavy quarks, target mass corrections, and higher twist terms
  - Order of perturbation theory
  - Errors on the data ➡ ***Only* error included!**
- Techniques have been developed to handle the last one
- The others require judgement and experience, but *are not* included in what are generally referred to as PDF errors.

Excellent review by J. Owens at CTEQ 2007 summer school,
http://www.phys.psu.edu/~cteq/schools/summer07/

Two recent examples...

PDF error examples

CTEQ, P. Nadolsky et al. '08



Inclusion of m_c, m_b suppresses F_2 at low $Q^2 \Rightarrow$ increase u,d to compensate

6-7% increase in LHC W, Z predictions

MSTW 2008 PDF release arXiv:0901.0002

- Run II inclusive jet data
- Gluon density decreased at x~0.1

M_H=170 GeV Higgs at Tevatron:

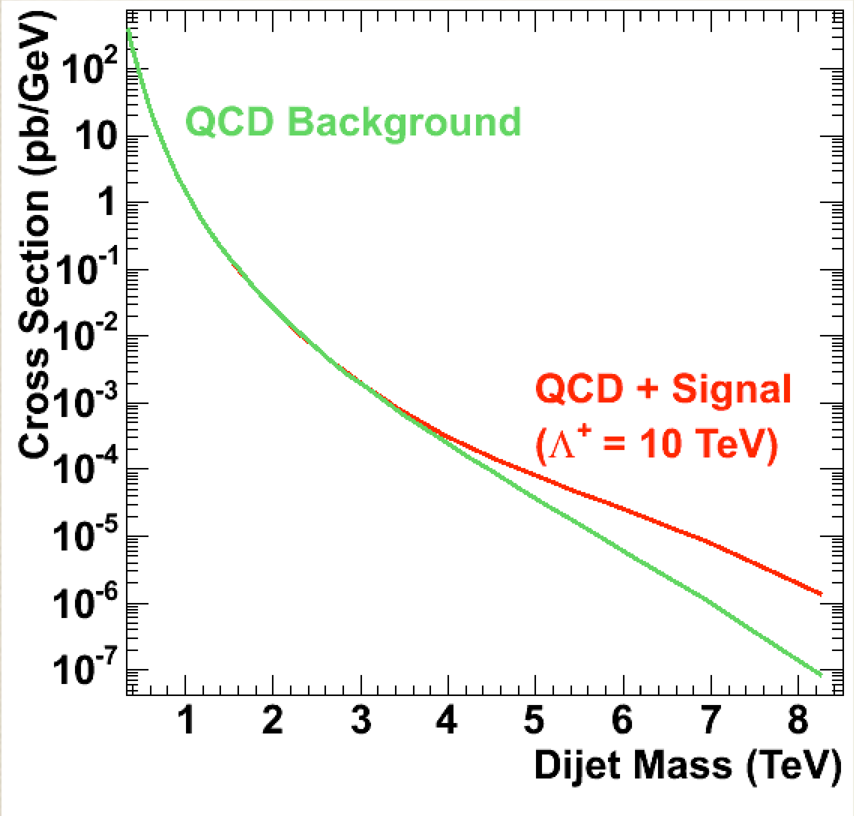
MRST 2001	MRST 2004	MRST 2006	MSTW 2008
0.3833	0.3988	0.3943	0.3444

Anastasiou, Boughezal, FP 0811.3458

~10-15% **decrease** in predicted cross section !
Previous 90% CL error: ±5%

Keep in mind for LHC applications...

Conclusions



from T. LeCompte, CTEQ 2007 summer school

- Goal of pQCD at the LHC: don't confuse these two lines
- Exact matrix elements or parton showers? Hard jets, angular correlations: MEs Soft/collinear emissions: PS
- NLO corrections 30% (qq) or 100% (gg) Quantitative descriptions of normalization, shapes require at least NLO
- Techniques exist for merging LO/NLO+PS
- NNLO needed for W, Z, H, PDFs+α_s
- Remember PDF errors *only* reflect experimental errors on used data sets!